Friedman-Stanley embedding of graphs in trees

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Abstract

Friedman and Stanley developed the notion of *Borel reducibility* and illustrated its use in comparing "classification problems" for some familiar classes of countable structures. Using known embeddings due to Lavrov, Mal'tsev, and others, they showed that undirected graphs, fields of fixed characteristic, and 2-step nilpotent groups lie on top. They descrubed embeddings of graphs in labeled trees and linear orderings, showing that those classes are also on top. We focus on the Friedman-Stanley embedding of graphs in labeled trees. For many embeddings, the fact that the embedding is $1-1$ on isomorphism types is explained by the existence of simple formulas that, uniformly, interpret the input structure in the output structure. For the embedding of graphs in labeled trees, Harrison-Trainor and Montalbán showed that this is not the case. Gonzalez and Rossegger showed that the embedding preserves Scott complexity. We refine this, showing that for an X-computable ordinal, the input graph $\mathcal{A}$ has an X-computable $\Pi_{\alpha+1}$ Scott sentence iff the output tree $T_{\mathcal{A}}$ does. Given a computable infinitary Scott sentence for $\mathcal{A}$, we use forcing to obtain one for $T_{\mathcal{A}}$.