

BEGINNINGS OF DEFINABILITY THEORY

A. L. SEMENOV, S. F. SOPRUNOV

The **main objective** of our talk is to draw attention to the theory of definability, which offers remarkable research prospects due to the fact that throughout the development of mathematical logic it has constantly been "out of focus" of the main attention of researchers. The beginnings – "launchs" of research occurred several times. Today we propose a re-launch based on a set of open problems in our talk and more – outside.

Among the **beginnings of the past**, we point to: the research of the 19th century a summit of which was the "Padoa method"; the "golden age" of mathematical logic in 1920-30s – the results of Gödel and Tarski, the last continued strong interest in definability for the rest of his life; 1959 – the Lars Svenonius theorem, which established the universality of Padoa method and opened the Erlangen program for the Geometry of Logic; homogeneous structures: 1964 – the theorem of Claude Frasnay, 1991 Simon Thomas' result, his hypothesis and followed research; 2000s – our research of non-homogeneous structures.

The report will provide **initial invariant definitions**: definability, definability space, definability lattice of structure. The relations between definability and decidability, hierarchies within the same definability space in terms of the number of arguments of the generators and the quantifier height are considered. The basic example and the first result on the definability lattice is the 5-element lattice for the order of rational numbers (Huntington – Frasnay); the effect of extending with a constant is impressive. The Svenonius theorem makes it possible to study definability lattices by constructing lattices of automorphism groups of elementary extensions of structures.

Results and open problems:

- The Rabin – Elgot problem of the existence of a maximal space with a decidable theory. The monadic case was solved by Soprunov. The elementary case is open.

- Thomas problem for homogeneous structures: prove the finiteness of the lattice.
- Quantifier hierarchy: find natural examples with depth 3, 4,...

Non-homogeneous structures:

- Discrete homogeneous: the lattice for integers with successor. Open questions on lattices: naturals with successor, infinite tree of degree 3, shifts on checkered paper.

- The order of integers: A fragment of the structure, the existence of elements outside the fragment.

LOMONOSOV MOSCOW STATE UNIVERSITY, MOSCOW (RUSSIA)

Email address: alexei.semenov@math.msu.ru