SPECTRAL THEORY OF LOCALLY FINITE GRAPHS

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A graph is called locally finite if degrees of all its vertices are finite. For a locally finite graph Γ and a field F, eigenvalues and their corresponding eigenfunctions of Γ over F are defined as eigenvalues and their corresponding eigenfunctions of adjacency matrix of the graph Γ over the field F, acting in the natural way on the vector space over F of all F-valued functions on the vertex set $V(\Gamma)$ of Γ. In the talk, a theory of eigenvalues and eigenfunctions of infinite locally finite connected graphs over fields is given. A special emphasis will be placed on the case of fields of characteristic 0. One of the consequences of the theory is that for an arbitrary infinite locally finite connected graph Γ, there are rational functions $R_v(x) \in \mathbb{Q}(x)$, $v \in V(\Gamma)$, where $R_w(x) \neq 0$ for some $w \in V(\Gamma)$, such that for any $\lambda \in \mathbb{C}$ which is not a pole of any of $R_v(x)$, $v \in V(\Gamma)$, and is not a zero of at least one of $R_v(x)$, $v \in V(\Gamma)$, the function $v \mapsto R_v(\lambda)$ is an eigenfunction of Γ over $\mathbb{C}$, corresponding to the eigenvalue λ. In particular, only algebraic numbers can be non-eigenvalues of Γ over $\mathbb{C}$.

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