

Decidable models in the theory of well-orderings

Sergey Goncharov

Sobolev Institute of Mathematics, Novosibirsk, Russia

e-mail: `s.s.goncharov@math.nsc.ru`

In this talk we present results on decidable (strongly constructive) models built from ordinal constructions, obtained within computable model theory [1]. The foundational results on the theory of well-orderings are due to Doner, Mostowski and Tarski [2], who proved the decidability of the elementary theory of every ordinal and of the theory of all ordinals, and established a number of model-theoretic results for these theories.

Using this technique together with computability theory, it was first shown, jointly with A.M. Petrenko, V.K. Kravtsov and E.I. Khlestova, that the prime model of the complete theory of any ordinal is decidable. This follows from the decidability of the theory [2], the computability of the set of complete formulas of any such theory, and the theorem on decidability of prime models [1]. As a consequence, we obtain decidable presentations of the ordinal ω^ω and of every ordinal $\lambda < \omega^\omega + \omega^\omega$.

Next, using the theory of recursive ordinals [3] and the elimination of quantifiers from [2], we prove the following result, which can be applied to inductive constructions of models on the basis of ordinals in order to study their strong constructivity effectively.

Theorem. Every computable ordinal has a strongly constructive presentation.

Finally, jointly with E. I. Khlestova, these results were extended to prime and saturated models of all complete extensions of the theory of all ordinals.

References

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- [3] H. Rogers, Jr. *Theory of Recursive Functions and Effective Computability*. McGraw-Hill, New York, 1967.