Title: Sequences of n-diagrams

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Abstract:

For a structure $\mathcal{A}$, the *n-diagram* is $D_n(\mathcal{A}) = D^c(\mathcal{A}) \cap \Sigma_n$. For some structures, the complete diagram is computable in the n-diagram, for some fixed n. In particular, this is true where we have effective elimination of quantifiers. We have the following general result.

Theorem: For a computable structure $\mathcal{A}$, and $n \geq 1$, the following are equivalent:

(1) for all $\mathcal{B} \cong \mathcal{A}$, $D^c(\mathcal{B}) \equiv_T D_n(\mathcal{B})$,

(2) there is a computable function d taking each finitary formula $\varphi(\overline{x})$ to an infinitary formula $\psi(\overline{x})$, where $\psi(\overline{x})$ is a c.e. disjunction of finitary Π_{n-1} formulas, with a fixed tuple of parameters, such that $\mathcal{A} \models \forall \overline{x}\, [\varphi(\overline{x} \leftrightarrow \psi(\overline{x})]$.

Example: If $\mathcal{B}$ is an ordering of type $\omega^N \cdot \eta$, $D^c(\mathcal{B}) \equiv_T D_{2N}(\mathcal{B})$.

Next, we consider possible sequences of degrees of n-diagrams for copies of a given structure. There is an obvious necessary condition on the sequence of n-diagrams—the $(n+1)$-diagram is c.e. in and above the n-diagram, uniformly in n. We show that under certain conditions, this is the only restriction.

Definition: An *ω-table* is a sequence $(C_n)_{n\in\omega}$ such that C_{n+1} is c.e. in and above C_n, uniformly in n.

Example: For any ω-table $(C_n)_{n\in\omega}$, there is an ordering $\mathcal{B}$ of type $\omega^\omega \cdot \eta$ such that for all n, $D_n(\mathcal{B}) \equiv_T C_n$.