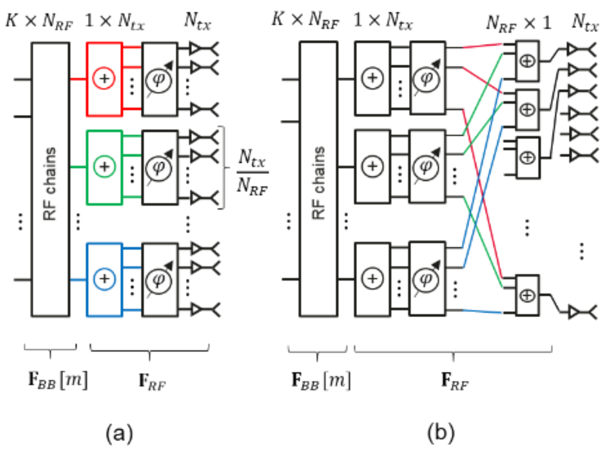
Sensing matrix optimization for Hybrid Beamforming

Hybrid beamforming (HBF) architecture is compromise between hardware complexity and performance with FD architecture [1]. Hardware complexity reduction is connected with smaller number of digital ports, while performance degradation is connected with smaller number of independent spatial beams radiated by antenna array. Maximum number of spatial degrees of freedom (DoF) is equal to number of antenna elements in FD architecture, for HBF architecture, is limited with number of RF chains which is smaller than number of physical antenna elements. Motivation example for HBF is following: antenna array has the same number of spatial beams as FD antenna array with same number of digital ports, while width of beams may be much smaller for HBF with efficient aperture utilization. Due to the effective aperture increasement HBF architecture with same number of digital ports as FD array may have higher angular resolution of beamforming providing better interference cancelation in multi-user scenario. Extra-large antenna arrays (ELAA) are considered now for future wireless communications and HBF allows to get acceptable hardware complexity for usage in practical systems.

Main types of HBF considered in literature is full connected (FC), partial connected (PC) [2]. In HBF antenna elements are connected with small set of ports for digital processing over analog network. This ports called digital ports. Parameters of analog network may be tuned by controllable elements like phase shifter (PS) that change the phase of input signal to given value. In case of PC architecture, antenna array is divided into sub-arrays, every sub-array is connected with single digital port and every antenna element in sub-array is connected with this port over PS. PC architecture has simplest hardware complexity – one PS per antenna element. In FC architecture every antenna element is connected over PS with every digital port, so number of PSs per antenna is equal to the number of digital ports. Additionally to PSs, every antenna element in FC array has power combiners that introduce unavoidable power losses and amplifiers must be used to compensate it.



1	9	17	25	33	41	49	57	65
2	10	18	26	34	42	50	58	66
3	11	19	27	35	43	51	59	67
4	12	20	28	36	44	52	60	68
5	13	21	29	37	45	53	61	69
6	14	22	30	38	46	54	62	70
7	15	23	31	39	47	55	63	71
8	16	24	32	40	48	56	64	72

(c)

Fig.1 HBF architectures: (a) - PC, (b) – FC, (c) – sub array splitting for PC

Channel estimation in case of HBF is quite different from full digital (FD) case. In uplink measurement, where user (UE) transmit pilot signal and Base Station (BS) antenna array receive this, signal from individual antenna element is available for processing. In HBF case, received signal available at digital ports is

$$r = F_{RF} h_S, \tag{1}$$

where $r \in \mathbb{C}^{N_{RF}}$ is received vector, $F_{RF} \in \mathbb{C}^{N_{RF} \times N_{BS}}$ is the analog network matrix, , $h_S \in \mathbb{C}^{N_{BS}}$ is vector received at antenna elements, N_{RF} is the number of digital ports, N_{BS} is the number of antenna elements at BS antenna. In HBF $N_{RF} \ll N_{BS}$ as main reason is reduction of ports for digital processing and vector h_X can be found up to arbitrary null-space vector after solving (1) as system of linear equations. Compressive Sensing (CS) methods allow to find unique solution of (1) by supposing that h_S has sparse representation in some set of vectors called dictionary. In case of wireless channels h_S is sparse in space of spatial frequencies, then received signal may represented like

$$r = F_{RF} F_{XY} h_K, \tag{2}$$

where $F_{XY} = F_X \otimes F_Y$ is Kronecker product dictionary for two-dimensional antenna array, $F_X \in \mathbb{C}^{N_X \times N_X}$, $F_Y \in \mathbb{C}^{N_Y \times N_Y}$ corresponding Discrete Fourier Transformation matrices, h_K sparse vector, $N_{BS} = N_X N_Y$. In CS naming convention F_{XY} is the dictionary matrix, F_{RF} is the measurement matrix and $F_{RF} F_{XY}$ is the sensing matrix. Consider for most interesting case of PC architecture for single time moment measureemtn. If $s_i, i = 1 \dots N_{RF}$ denote set of non-zero indexes for i-th row of matrix F_{PC}, then $s_i \cap s_j = \emptyset, \forall i \neq j$ and $\cup_i s_i = 1 \dots N_{BS}$, $|s_i| = \frac{N_{BS}}{N_{RF}}, \forall i$ and $\left|[F_{PC}]_{jk}\right| = \sqrt{N_{RF}/N_{BS}}$ for non-zero components of F_{PC}, note that $\frac{N_{BS}}{N_{RF}}$ is the number of antenna elements in sub-array. Example for sub-array splitting from Fig.1(c) is shown at Fig. 2, where different colors denote assignment to different digital ports, every column has single non-zero elements and non-zero matrix elements have magnitude $\left|[F_{PC}]_{jk}\right| = 1/3$ as sub-array has 9 elements i.e. $|s_i| = 9$.

Antenna element index

Digital port index

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2																											
3																											
4																											

Fig. 2 Example of analog network matrix for PC architecture shown at Fig. 1(c),
$s_1 = \{1,2,3,7,8,9,13,14,15\}, s_2 = \{4,5,6,10,11,12,16,17,18\}, \dots$

For multiple time moment r is concatenation of vectors from single time moments and F_{PC} columns are concatenation of columns of matrices from single time moments. If $N_T N_{RF} \geq N_{BS}$, where N_T is number of time moments for measurement, problem (1) is LS problem and F_{RF} may be designed non-singular by tuning phases of non-zero components. In case of small number or single time moments sensing matrix $B = F_{RF} F_{XY}$ may be designed optimal in sense of CS based criteria. One of tractable criteria for sensing matrix optimization is incoherence minimization, where incoherence is defined for matrix with unit norm columns as

$$\mu_B = \max_{i \neq j} \left|\langle b_i, b_j \rangle\right|.$$

Incoherence cannot be less than fundamental bound called Welch bound. In case of HBF, columns of sensing matrix may have different norms so additional criteria for measurement matrix optimization like equal norm columns must be considered.

Wireless channel may change due to the user movement, in process of measurement during multiple time moments, so measurement process must be fulfilled in short time. Different frequencies may be used for measurement simultaneously, if number of time moments is not enough for channel estimation with acceptable quality. Dictionary is then $F_{FXY} = F_F \otimes F_X \otimes F_Y$, $F_F \in \mathbb{C}^{N_F \times N_F}$, N_F is the number of subcarriers used by wireless system, wireless channel also has sparse representation in this dictionary. Measurement matrix is then

$$F_{RF} = \begin{bmatrix} F_1^T & F_2^T & \dots & F_{N_T}^T \end{bmatrix}^T, F_i = P_i I_{N_F} \otimes F_{PC,i} ,$$

where $\boldsymbol{P_i} \in \mathbb{R}^{round(a_f N_F) \times N_F}$ is row selection matrix connected with frequencies used at i-th moment of time, a_f is a part of frequencies that used at every moment of measurement, sets of frequencies used in different time moments may have intersections, set of non-zero column indexes for $\boldsymbol{P_i}$ defines F_i set of frequencies, $\boldsymbol{F_{PC,i}}$ is a single moment measurement matrix described above. Example of block structure for measurement matrix with three time moments is shown at Fig. 3. Every row set in $\boldsymbol{F_{RF}}$ with indexes $1 + (n-1)N_{RF} \dots nN_{RF}$.

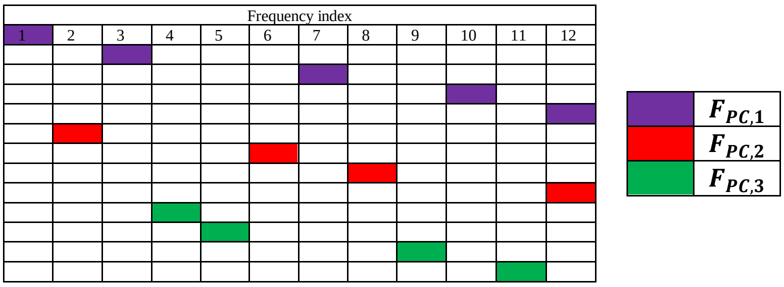


Fig. 3. Block structure for measurement matrix $\boldsymbol{F_{RF}}$ with three measurement time moments. Every block has $\boldsymbol{N_{RF}} \times \boldsymbol{N_{BS}}$ size.

Gram matrix is then

$$\boldsymbol{G} = \boldsymbol{F_{FXY}^H F_{RF}^H F_{RF} F_{FXY}}$$

and optimization problem is then

$$\begin{gathered} \min \|\boldsymbol{G} - \boldsymbol{\alpha I}\|_F^2 \\ s.t.\ |F_i| = \alpha_f, i = 1 \dots N_T; \\ s_i \cap s_j = \emptyset, \forall i \neq j\ ; \cup_i s_i = 1 \dots N_{BS}; |s_i| = \frac{N_{BS}}{N_{RF}}, \forall i \\ \left|[F_{PC,i}]_{jk}\right| = \sqrt{N_{RF}/N_{BS}} \end{gathered} \qquad \text{(P1)}$$

where α is empirical term responsible for equal norms for diagonal elements of Gram matrix. Both incoherence minimization and column norms equalization are included into (P1). In literature [3] exists solutions for optimization sensing matrices for single frequency case, but joint optimization of frequency set and analog network matrix is not presented. It interesting to test acquired problem solution with another sets of frequencies i.e. fix $F_{PC,i}$, $i = 1 \dots N_T$ and change P_i to another sets. If problem has no high sensitivity to frequency sets then solution may be used for other users with different frequency sets for simultaneous channel estimation. Feasible numerical parameters are $N_X = 16{,}32$, $N_Y = 32{,}64$, $N_F = 128{,}256$, $N_T = 2{,}3$, $N_{RF} = 16{,}32{,}64$, $|F_i| = 0.05 \dots 0.25 N_F$, sub-array geometries are $1 \times 8, 2 \times 8, 4 \times 4, 4 \times 8$. Optimization problem (P1) statement may be changed if better for simultaneous incoherence and column norm equalization exists.

We would greatly appreciate any insights, recommendations, or innovative approaches that could assist in addressing this challenge. Your expertise and contributions could significantly enhance our collaborative efforts, paving the way for deeper exploration and fruitful future partnerships.

References

[1] S. Park, A. Alkhateeb and R. W. Heath, "Dynamic Subarrays for Hybrid Precoding in Wideband mmWave MIMO Systems," in IEEE Transactions on Wireless Communications, vol. 16, no. 5, pp. 2907-2920, May 2017, doi: 10.1109/TWC.2017.2671869

[2] L. Zhang, L. Gui, Q. Qin and Y. Tu, "Adaptively-Connected Structure for Hybrid Precoding in Multi-User Massive MIMO Systems," 2018 IEEE 29th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Bologna, Italy, 2018, pp. 1-7, doi: 10.1109/PIMRC.2018.8581003

[3] Ardah, K., Haardt, M., Liu, T., Matter, F., Pesavento, M., & Pfetsch, M. E. (2022). Recovery under Side Constraints. In Compressed Sensing in Information Processing (pp. 213-246). Cham: Springer International Publishing.