ABOUT COHOMOGENEITY ONE ALMOST COMPLEX STRUCTURES ON THE $S^2 \times S^4$

N. Daurtseva

(Kemerovo State University, Kemerovo, Russia)

E-mail address: `natali0112@ngs.ru`

The action of a group G on a manifold M is said to be *cohomogeneity one* if the orbit space M/G is one-dimensional. M is called *an interval cohomogeneity one manifold* if the orbit space M/G is a closed interval $[0, T] \subset \mathbb{R}$. Such a manifold is determined by its group diagram $G \supset K^{\pm} \supset K$. Here K is called a *principal isotropy subgroup* and $K^{\pm}$ are *non-principal isotropy subgroups.* These groups satisfy the condition $K^{\pm}/K \simeq S^{l_{\pm}}$. The open set $M^* \subset M$ corresponding to the interior of M/G is diffeomorphic to $(0, T) \times G/K$, and $G/K^{\pm}$ are non-principal orbits corresponding to the boundary points of M/G. Conversely any collection of compact groups $G \supset K^{\pm} \supset K$, with $K^{\pm}/K \simeq S^{l_{\pm}}$ determines an interval cohomogeneity one manifold.

At [1, 2] authors classified all possible group diagrams of cohomogeneity one nearly Kähler 6-manifolds. The case of $S^2 \times S^4$, with group diagram $SU(2) \times SU(2) \supset U(1) \times SU(2), U(1) \times SU(2) \supset \Delta U(1)$ was overlooked, but later was specified at [3] by Foscolo L. and Haskins M. In [3] authors have proven the existence of exotic nearly Kähler structures on S^6 and $S^3 \times S^3$ which are inhomogeneous but of cohomogeneity one. For $S^2 \times S^4$ was conjectured that it carries no cohomogeneity one nearly Kähler structure.

The $S^2 \times S^4$ is a special manifold for a number of reasons. Firstly this is in list of almost complex even-dimensional spheres products [4], and the unique one with non almost complex multiplier S^4. It is diffeomorphicaly embeddable in $\mathbb{R}^7$ and inherits Cayley structure. The Cayley structure is practically unique example of the almost complex structure on $S^2 \times S^4$, and it is not $SU(2) \times SU(2)$-cohomogeneity one structure. The questions about existence of nearly Kähler or complex structures on $S^2 \times S^4$ are open.

At the talk I will give new examples of cohomogeneity one almost complex structures with some additional properties on $S^2 \times S^4$.

References

[1] F. Podestà, A. Spiro. Six-dimensional nearly Kähler manifolds of cohomogeneity one // J. Geom. Phys., **60**:2 (2010), 156–164.

[2] F. Podestà, A. Spiro. Six-dimensional nearly Kähler manifolds of cohomogeneity one (II) // Comm. Math. Phys., **312**:2 (2012), 477–500.

[3] L. Foscolo, M. Haskins. New G_2-holonomy cones and exotic nearly Kahler structures on S^6 and $S^3 \times S^3$ // Ann. Math., **185**:1 (2017), 59–130.

[4] B. Datta, S. Subramanian. Nonexistence of almost complex structures on product of even-dimensional spheres // Top. and App., **36** (1990), 39–42