



LABORATORY OF TOPOLOGY AND DYNAMICS,
NOVOSIBIRSK STATE UNIVERSITY

and

SOBOLEV INSTITUTE OF MATHEMATICS

Conference "Integrable Systems and Their Applications"*

05-11 August 2019, Baykalsk

*The conference is supported by the Laboratory of Topology and Dynamics,
Novosibirsk State University (contract no. 14.Y26.31.0025 with the Ministry of
Education and Science of the Russian Federation)

Program committee

Andrey Mironov (Sobolev Institute of Mathematics and Novosibirsk State University, Russia)

Iskander Taimanov (Sobolev Institute of Mathematics and Novosibirsk State University, Russia)

Youjin Zhang (Tsinghua University, China)

Jian Zhou (Tsinghua University, China)

Conference Schedule

Tuesday, August 6 (Conference hall)

10:00–10:05 *Opening of the conference*

10:05–10:50 **J. Zhou**, *Dualities and integrable systems*

10:55–11:40 **N. Tyurin**, *Monotonic Lagrangian tori of non standard type in toric manifolds*

11:40–12:00 *Coffee break (20 minutes)*

12:00–12:45 **Sh. Guo**, *Mathematical approaches to the BCOV's holomorphic anomaly*

Wednesday, August 7 (Conference hall)

10:00–10:45 **Y. Zhang**, *Special Cubic Hodge Integrals and the Fractional Volterra Hierarchy*

10:50–11:35 **A. Mironov**, *Baker–Akhiezer modules on algebraic varieties*

11:35–11:55 *Coffee break (20 minutes)*

11:55–12:40 **Zh. Zong**, *Remodeling Conjecture and its applications*

Thursday, August 8 (Conference hall)

10:00–10:45 **D. Zuo**, *Frobenius manifolds and a new class of Extended affine Weyl groups of A-type*

10:50–11:35 **M. Pavlov**, *Integrability of new class of hyperbolic two-dimensional linear equations of second order*

11:35–11:55 *Coffee break (20 minutes)*

11:55–12:40 **N. Daurtseva**, *About cohomogeneity one almost complex structures on the $S^2 \times S^4$*

Friday, August 9 (Discussion day)

19:30 *Social dinner*

Conference "Integrable Systems and Their Applications"

Saturday, August 10 (Conference hall)

10:00–10:45 **H. Ma**, *Ruh–Vilms theorems for minimal Lagrangian surfaces in $\mathbb{C}P^2$*

10:50–11:35 **A. Zheglov**, *The theory of commuting differential operators: the state of art*

11:35–11:55 *Coffee break (20 minutes)*

11:55–12:40 **S. Agapov**, *On first integrals of Hamiltonian systems on the 2-torus*

Poster session (Wednesday, August 7, 11:35–11:55)

G. Mauleshova, *On one-point commuting difference operators of rank 1*

M. Yermentay, *On minimal isotropic surfaces in $\mathbb{C}P^3$*

**ON FIRST INTEGRALS OF HAMILTONIAN SYSTEMS ON THE
2-TORUS**

SERGEI AGAPOV

Novosibirsk State University, Novosibirsk, Russia
Sobolev Institute of Mathematics, Novosibirsk, Russia
agapov@math.nsc.ru

We consider Hamiltonian systems related to geodesic flows (including magnetic ones) and natural mechanical systems on the 2-torus and study the question of their integrability. Generally speaking, this problem can be reduced to the search for an additional first integral which is independent on the Hamiltonian. The different questions related to local and global existence of such integrals, polynomial or rational in momenta, will be discussed.

This talk is based on joint results with M. Bialy, A.E. Mironov, A.A. Valyuzhenich.

ABOUT COHOMOGENEITY ONE ALMOST COMPLEX
STRUCTURES ON THE $S^2 \times S^4$

NATALIYA DAURTSEVA

Kemerovo State University, Kemerovo, Russia
natali0112@ngs.ru

The action of a group G on a manifold M is said to be *cohomogeneity one* if the orbit space M/G is one-dimensional. M is called an *interval cohomogeneity one manifold* if the orbit space M/G is a closed interval $[0, T] \subset \mathbb{R}$. Such a manifold is determined by its group diagram $G \supset K^\pm \supset K$. Here K is called a *principal isotropy subgroup* and K^\pm are *non-principal isotropy subgroups*. These groups satisfy the condition $K^\pm/K \simeq S^{l^\pm}$. The open set $M^* \subset M$ corresponding to the interior of M/G is diffeomorphic to $(0, T) \times G/K$, and G/K^\pm are non-principal orbits corresponding to the boundary points of M/G . Conversely any collection of compact groups $G \supset K^\pm \supset K$, with $K^\pm/K \simeq S^{l^\pm}$ determines an interval cohomogeneity one manifold.

At [1, 2] authors classified all possible group diagrams of cohomogeneity one nearly Kähler 6-manifolds. The case of $S^2 \times S^4$, with group diagram $SU(2) \times SU(2) \supset U(1) \times SU(2), U(1) \times SU(2) \supset \Delta U(1)$ was overlooked, but later was specified at [3] by Foscolo L. and Haskins M. In [3] authors have proven the existence of exotic nearly Kähler structures on S^6 and $S^3 \times S^3$ which are inhomogeneous but of cohomogeneity one. For $S^2 \times S^4$ was conjectured that it carries no cohomogeneity one nearly Kähler structure.

The $S^2 \times S^4$ is a special manifold for a number of reasons. Firstly this is in list of almost complex even-dimensional spheres products [4], and the unique one with non almost complex multiplier S^4 . It is diffeomorphically embeddable in \mathbb{R}^7 and inherits Cayley structure. The Cayley structure is practically unique example of the almost complex structure on $S^2 \times S^4$, and it is not $SU(2) \times SU(2)$ -cohomogeneity one structure. The questions about existence of nearly Kähler or complex structures on $S^2 \times S^4$ are open.

At the talk I will give new examples of cohomogeneity one almost complex structures with some additional properties on $S^2 \times S^4$.

REFERENCES

- [1] F. Podestà, A. Spiro, Six-dimensional nearly Kähler manifolds of cohomogeneity one // J. Geom. Phys., **60**:2 (2010), 156–164.
- [2] F. Podestà, A. Spiro. Six-dimensional nearly Kähler manifolds of cohomogeneity one (II) // Comm. Math. Phys., **312**:2 (2012), 477–500.
- [3] L. Foscolo, M. Haskins. New G_2 -holonomy cones and exotic nearly Kahler structures on S^6 and $S^3 \times S^3$ // Ann. Math., **185**:1 (2017), 59–130.
- [4] B. Datta, S. Subramanian. Nonexistence of almost complex structures on product of even-dimensional spheres // Top. and App., **36** (1990), 39–42.

**MATHEMATICAL APPROACHES TO THE BCOV'S
HOLOMORPHIC ANOMALY**

SHUAI GUO

Peking University, Beijing, China
guoshuai@math.pku.edu.cn

During the last two decades, it has been a central problem to compute the Gromov-Witten (GW) invariants of Calabi-Yau threefolds, in both geometry and physics. In this talk, we will discuss the recent mathematical approaches to the all genera Gromov-Witten potential functions of the quintic threefolds. We will also discuss about the possible generalization of our method.

**RUH-VILMS THEOREMS FOR MINIMAL LAGRANGIAN
SURFACES IN $\mathbb{C}P^2$**

HUI MA

Tsinghua University, Beijing, China
ma-h@mail.tsinghua.edu.cn

In this talk, we investigate surfaces in $\mathbb{C}P^2$ without complex points and characterize minimal Lagrangian surfaces by Ruh-Vilms type theorems.

**ON ONE-POINT COMMUTING DIFFERENCE OPERATORS OF
RANK 1**

GULNARA MAULESHOVA

Novosibirsk State University, Novosibirsk, Russia
Sobolev Institute of Mathematics, Novosibirsk, Russia
mauleshova@math.nsc.ru

We construct solutions to the difference-differential equation that are associated with one-point commuting difference operators of rank 1 in the case of spectral curves of genus 1.

BAKER–AKHIEZER MODULES ON ALGEBRAIC VARIETIES

ANDREY MIRONOV

Novosibirsk State University, Novosibirsk, Russia
Sobolev Institute of Mathematics, Novosibirsk, Russia
mironov@math.nsc.ru

In the talk we discuss Baker–Akhiezer modules on algebraic varieties introduced by A. Nakayashiki. This construction gives examples of commuting partial differential operators with matrix coefficients.

**INTEGRABILITY OF NEW CLASS OF HYPERBOLIC
TWO–DIMENSIONAL LINEAR EQUATIONS OF SECOND ORDER.**

MAXIM PAVLOV

P.N. Lebedev Physical Institute of the Russian Academy of Sciences, Moscow,
Russia
m.v.pavlov@lboro.ac.uk

We consider a linear two–dimensional hyperbolic equation of second order, whose coefficients are polynomials with respect to one of independent variables. We show that this equation possesses infinitely many particular solutions, determined by solutions of ordinary differential equations.

**MONOTONIC LAGRANGIAN TORI OF NON STANDARD TYPE
IN TORIC MANIFOLDS**

NIKOLAY TYURIN

Joint Institute for Nuclear Research, Dubna, Russia
Higher School of Economics, Moscow, Russia
ntyurin@theor.jinr.ru

In recent papers we constructed examples of non standard Lagrangian tori in compact simply connected toric symplectic manifolds. Using new "pseudotoric" technique one explained the appearance of non standard lagrangian tori of Chekanov type and proposed a topological obstruction which separates them from the standard one. In the talk we construct non standard tori which satisfy the Bohr–Sommerfeld condition with respect to the anticanonical class. Then we prove that if it exists a standard monotonic lagrangian torus in smooth simply connected toric Fano variety equipped with canonical symplectic form then it must exist monotonic lagrangian torus of the Chekanov type.

ON MINIMAL ISOTROPIC SURFACES IN \mathbb{CP}^3

MEIRAMGUL YERMENTAY

Novosibirsk State University, Novosibirsk, Russia
Sobolev Institute of Mathematics, Novosibirsk, Russia
ermentay.m@gmail.com

In this work, we constructed a family of minimal isotropic tori and Klein bottles in \mathbb{CP}^3 in terms of elementary functions.

**SPECIAL CUBIC HODGE INTEGRALS AND THE FRACTIONAL
VOLTERRA HIERARCHY**

YOUJIN ZHANG

Tsinghua University, Beijing, China
youjin@mail.tsinghua.edu.cn

We show that the generating function of cubic Hodge integrals satisfying the local Calabi–Yau condition is the tau function of a particular solution of an integrable hierarchy called the fractional Volterra hierarchy. This integrable hierarchy is a certain generalization of the Volterra lattice hierarchy (also called the discrete KdV hierarchy) which is well known in the theory of nonlinear integrable systems. The talk is based on joint work with Si–Qi Liu, Di Yang and Chunhui Zhou.

**THE THEORY OF COMMUTING DIFFERENTIAL OPERATORS:
THE STATE OF ART**

ALEXANDER ZHEGLOV

Moscow State University, Moscow, Russia
youjin@mail.tsinghua.edu.cn

The classification problem of commuting differential operators is a highly non-trivial task, which, being solved in complete generality, would lead to resolution of many other open questions in various branches of mathematics. Even in the well known case of ordinary differential operators there are still many open questions. In this talk we will discuss recent improvements of this theory.

DUALITIES AND INTEGRABLE SYSTEMS

JIAN ZHOU

Tsinghua University, Beijing, China
jianzhou@mail.tsinghua.edu.cn

In string theory many different models have been proposed. Because they are supposed to describe the same world, physicists have proposed a notion of duality which means the equivalence of different models. In this talk we will explain a proposal to understand such dualities based on the relationship with integrable systems.

REMODELING CONJECTURE AND ITS APPLICATIONS

ZHENGYU ZONG

Tsinghua University, Beijing, China
zyzong@mail.tsinghua.edu.cn

Based on the work of Eynard-Orantin and Marino, the Remodeling Conjecture was proposed in the papers of Bouchard-Klemm- Marino-Pasquetti in 2007 and 2008. The Remodeling Conjecture can be viewed as an all genus mirror symmetry for toric Calabi-Yau 3-orbifolds. It relates the higher genus open Gromov-Witten potential of a toric Calabi-Yau 3-orbifold to the higher genus B-model potential which is obtained by applying the topological recursion on the mirror curve.

**FROBENIUS MANIFOLDS AND A NEW CLASS OF EXTENDED
AFFINE WEYL GROUPS OF A-TYPE**

DAFENG ZUO

University of Science and Technology of China, Hefei, China
dfzuo@ustc.edu.cn

We present a new class of extended affine Weyl groups $\widetilde{W}^{(k,k+1)}(A_l)$ for $1 \leq k < l$ and obtain an analogue of Chevalley-type theorem for their invariants. We further show the existence of Frobenius manifold structures on the orbit spaces of $\widetilde{W}^{(k,k+1)}(A_l)$ and also construct Landau-Ginzburg superpotentials for these Frobenius manifold structures.