RECENT PROGRESS IN THE STUDY OF BOUNDEDNESS OF THE CLASSICAL OPERATORS OF REAL ANALYSIS IN GENERAL MORREY-TYPE SPACES

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Let $0 < p$, $\theta \leq \infty$ and let w be a non-negative measurable function on $(0, \infty)$. We denote by $LM_{p\theta,w}$, $GM_{p\theta,w}$, the local Morrey-type spaces, the global Morrey-type spaces respectively, which are the spaces of all functions $f \in L_p^{loc}(\mathbb{R}^n)$ with finite quasi-norms

$$\left\| w(r) \|f\|_{L_p(B_r)} \right\|_{L_\theta(0,\infty)}, \quad \sup_{x \in \mathbb{R}^n} \|f(x + \cdot)\|_{LM_{p\theta,w}}$$

respectively. (Here B_r is the ball of radius r centered at the origin.) For $w(r) = r^{-\frac{\lambda}{p}}$ with $0 < \lambda < n$ the spaces $GM_{p\theta,w}$ were introduced by C. Morrey in 1938 and appeared to be quite useful in various problems in the theory of partial differential equations.

A survey will be given of recent results in which, for a certain range of the parameters, necessary and sufficient conditions are established ensuring boundedness of the maximal operator, fractional maximal operator, Riesz potential and genuine singular integrals as operators from one Morrey-type space to another one. Compared with the case of weighted L_p-spaces there are much more open problems which will also be under discussion.