STABILITY PROPERTIES OF LINEAR NONAUTONOMOUS SECOND-ORDER DIFFERENTIAL EQUATIONS

Hatvani L.

Bolyai Institute, University of Szeged, Szeged, Hungary;
hatvani@math.u-szeged.hu

We consider the differential equation

$$x'' + a^2(t)x = 0,$$

which describes the oscillation of a material point under the action of a restoring force with varying elasticity coefficient. We are interested in conditions of stability and instability of the equilibrium position. In this topic the class of step function coefficients $a : [0, \infty) \to (0, \infty)$ plays an important role. To study this case we use probabilistic methods assuming that the sequence $\{a_k\}_{k=1}^{\infty}$ of heights of the steps in the step function a is given, but the sequence $\{t_k\}_{k=1}^{\infty}$ of the points of jumps is random; namely, differences $t_k - t_{k-1}$ are positive, independent, not necessarily identically distributed random variables. We investigate two fundamental cases.

1. The sequence $\{a_k\}_{k=1}^{\infty}$ is increasing and tends to ∞. It will find out that for a very wide class of random variables we have $\lim_{t\to\infty} x(t) = 0$ for all solutions of the equation (stability).

2. The sequence $\{a_k\}_{k=1}^{\infty}$ consists of two positive numbers near each others and it is periodic; moreover, the differences $t_k - t_{k-1}$ are independent, identically distributed random variables (Hill–Meissner equation, the “problem of swinging”). In this case we give conditions for the characteristic function of $t_k - t_{k-1}$ guaranteeing that the expected values of the total mechanical energy tend to ∞ as $k \to \infty$ (instability).