A LINEARIZED INVERSE PROBLEM FOR THE DIRICHLET-TO-NEUMANN MAP ON DIFFERENTIAL FORMS

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For a compact n-dimensional Riemannian manifold (M, g) with boundary $i : \partial M \subset M$, the Dirichlet-to-Neumann (DN) map $\Lambda_g : \Omega^k(\partial M) \to \Omega^{n-k-1}(\partial M)$ is defined on exterior differential forms by $\Lambda_g\varphi = i^*(\star d\omega)$, where ω solves the boundary value problem $\Delta\omega = 0$, $i^*\omega = \varphi$, $i^*\delta\omega = 0$. For a symmetric second rank tensor field h on M, let $\dot\Lambda_h = d\Lambda_{g+th}/dt|_{t=0}$ be the Gateaux derivative of the DN map in the direction h. We study the question: for a given (M, g), how large is the subspace of tensor fields h satisfying $\dot\Lambda_h = 0$? Potential tensor fields belong to the subspace since the DN map is invariant under isomeries fixing the boundary. For a manifold of an even dimension n, the DN map on $(n/2 - 1)$-forms is conformally invariant, therefore spherical tensor fields belong to the subspace in the case of $k = n/2 - 1$. The manifold is said to be Ω^k-rigid if there is no other h satisfying $\dot\Lambda_h = 0$. We prove that the Ω^k-rigidity is equivalent to the density of the range of some bilinear form on the space $\mathcal{H}^{k+1}_{ex}(M)$ of exact harmonic fields.