



The Use of Finite Field GF(256) in the Performance Primitives Intel® IPP

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VCSD/CIP/IPP

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Agenda

- Short IPP review
- GF(256) operations being in focus
- Methods for implementation operations over GF(256)
- IPP implementation of GF(256) operations
- Some of performance results

IPP. Short Review

Main metrics:

- **16** domains and **10000** functions
- high degree of **optimization**
- running on **wide range** Intel® CPUs
- **popular OS** (Windows, Linux, Mac OS)
- a lot of **samples** of usage
- developed **Q&A** infrastructure
- a lot of **customers** (40K licenses)

Factors of success:

- understanding of IA architectures
- architecture designer's support, including Instruction Set enhancement

Technical feature:

most of IPP functions are targeted on single and double floating point and integer data types

IPP customers:



Samples of Applications over GF(2⁸)

- protected version of AES
- RS decoder

	AES	RS decoder
main operation	multiplicative inversion $Z = X^{-1}$	polynomial value $P(X) = p_0 + p_1 \cdot X + \dots + p_{n-1} \cdot X^{n-1}$
basic operation	GF(2 ⁸) multiplications $Z = X \cdot Y, \quad Z = X \cdot C$	GF(2 ⁸) multiplication $Z = X \cdot Y$

Operation Specific – vector Mode

AES:

$\left. \begin{array}{l} \text{SubBytes()} \\ \text{ShiftRows()} \\ \text{MixColumns()} \\ \text{AddRoundKey()} \end{array} \right\} F_i(b_j),$
 $j=0, \dots, 15$

RS decoder:

$\left. \begin{array}{l} \text{syndromes error} \\ \text{locations error} \\ \text{values} \end{array} \right\} P_i(x_j), x_j$
 – fixed values

Additional Code Designer's Problem:

no instruction level support for GF multiplication

Implementation of GF(2⁸) Operations

1) **Table Lookup.** Any necessary table (**mulTbl**, **logTbl**, **alogTbl**, **invTbl** and etc.) can be pre-computed in advance and used in application.

multiplication:

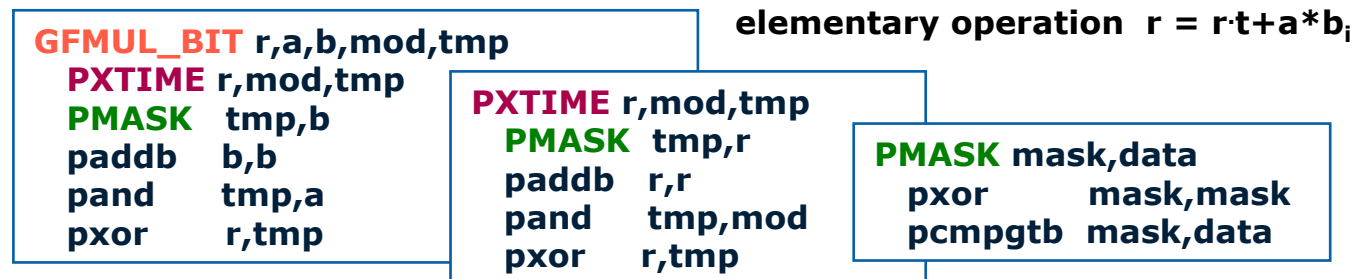
$z = x \cdot y \Leftrightarrow \text{mulTbl}[x][y]$ or

$z = x \cdot y \Leftrightarrow \text{alogTbl}[(\text{logTbl}[x] + \text{logTbl}[y]) \bmod 255]$

inversion:

$z = x^{-1} \Leftrightarrow \text{invTbl}[x]$

2) **Direct Computation.** Streaming SIMD Extension2 (SSE2) offers enough for direct GF(2⁸) multiplication. The **performance is comparable** with Lookup Method. But using SSE2-based multiplication for **inversion** looks **inefficient**.



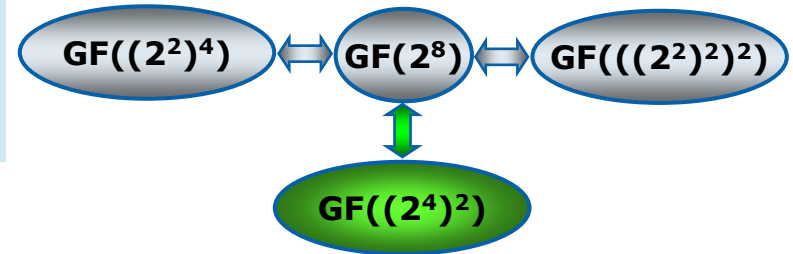
3) **Composite Fields.** It's known that GF(2^{nm}) is **isomorphic** to GF((2ⁿ)^m) but their arithmetic **complexity differ** and depends on choice of **n** and **m** and on constructing polynomials.

GF(2⁸) Operations. Composite Field Approach

Our choice:
Ground field:
Extension:

H: $\text{GF}(2^8) \Leftrightarrow \text{GF}((2^4)^2)$
 $\text{GF}(2^4)$, $q()=u^4+u+1$
 $\text{GF}((2^4)^2)$, $p=t^2+t+L$

$$8=2*2*2=2*4=4*2$$



multiplication $z=x \cdot y$

$$\begin{aligned} z_0 &= x_0 \cdot y_0 + x_1 \cdot y_1 \cdot L \\ z_1 &= x_0 \cdot y_1 + x_1 \cdot (y_0 + y_1) \end{aligned}$$

inversion $z=x^{-1}$

$$\begin{aligned} z_0 &= (x_0 + x_1) \cdot d^{-1} \\ z_1 &= x_1 \cdot d^{-1} \\ d &= x_0 \cdot (x_0 + x_1) + L \cdot x_1^2 \end{aligned}$$

arbitrary L

$$\{9, B, D, E\}$$

where $H(x)=x_0+x_1t$, $H(y)=y_0+y_1t$, $H(z)=z_0+z_1t$
and x_i, y_i, z_i belongs $\text{GF}(2^4)$

Composite Field are frequently used by HW designers

Implementation of GF(2⁴) Operations



➡ **SSSE3** ➡ **PSHUFB A,B ;;** [a_{b0},a_{b1},...,a_{b15}]

PLOOKUP dst,src,TBL
movdqa dst, TBL
pshufb dst, src

multiplication

PLOOKUP xmm2, xmm0, logTbl
PLOOKUP xmm3, xmm1, logTbl
paddb xmm2, xmm3
PLOOKUP xmm3, xmm2, alogTbl

															logTbl	
C0	0	1	4	2	8	5	A	3	E	9	7	6	D	B	C	
1	2	4	8	3	6	C	B	5	A	7	E	F	D	9	1	
															alogTbl	

multiplication inversion

LOOKUP xmm1, xmm0, invTbl

															invTbl	
0	1	9	E	D	B	7	6	F	2	C	5	A	4	3	8	

const. multiplication

LOOKUP xmm1, xmm0, mul9

(*9)

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

square

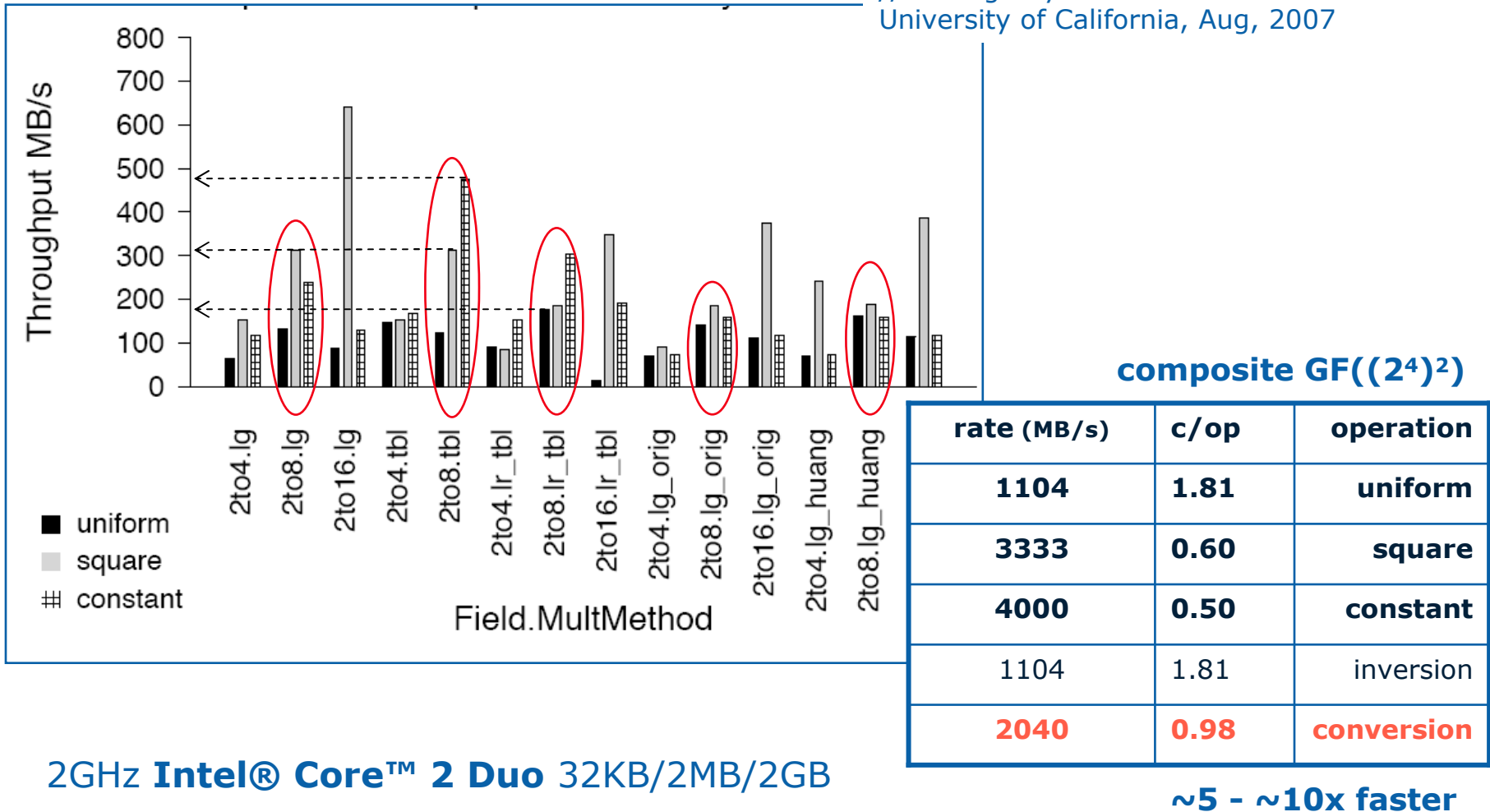
LOOKUP xmm1, xmm0, sqrTbl

															sqrTbl	
0	1	4	5	3	2	7	6	C	D	8	9	F	E	B	A	

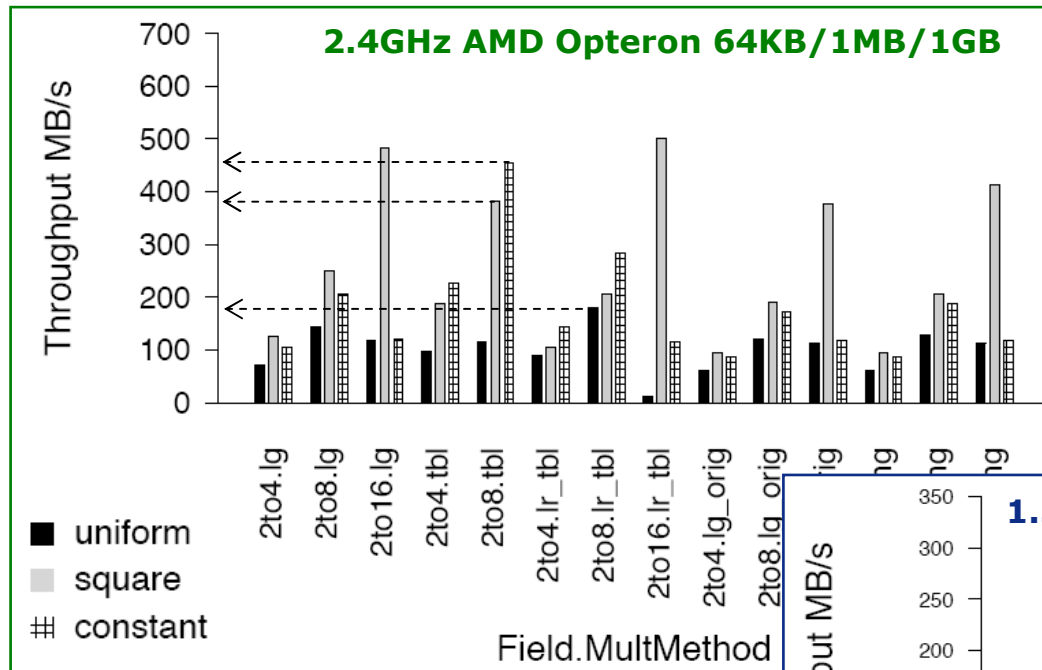
Basic Operations Performance. IA

Analysis and Construction of Galois Fields for Efficient Storage Reliability. Tech. Report UCSC-SSRC-07-09

// Storage Systems Research Center &
University of California, Aug, 2007



Basic Operations Performance. AMD, PPC



2009, SSE5, AMD:

pperm dst, src1,src2, ctr

// for (i=0; i<15; i++)

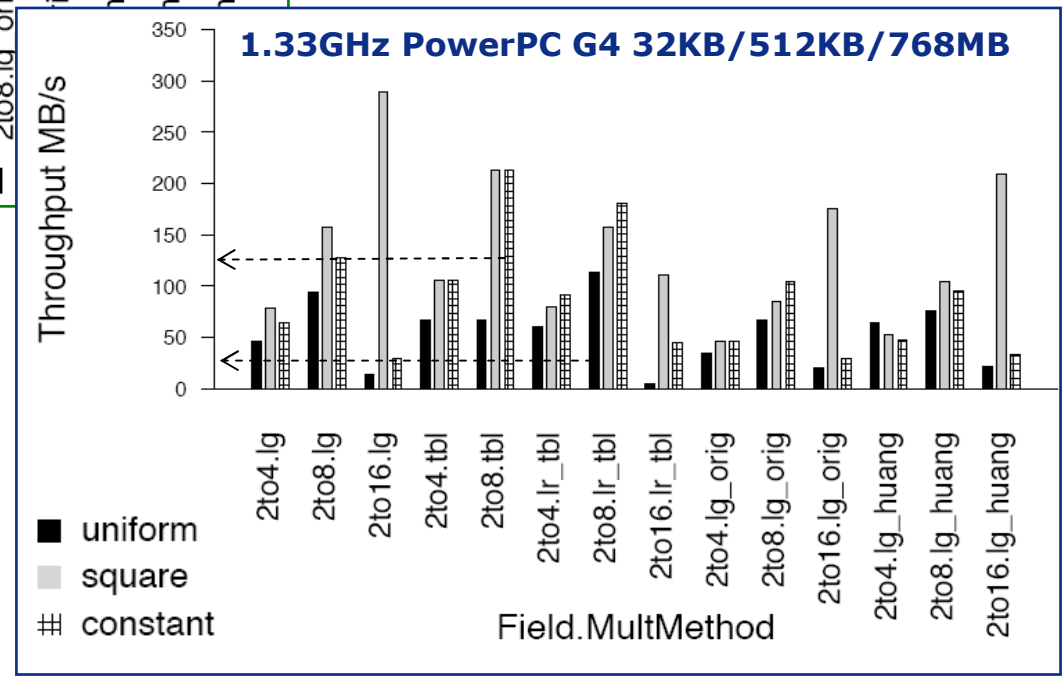
// dst[i] = ctr[i].op (src1|src2)ctr[i].sel

PPC G4:

vperm dst, src1,src2, ctr

// for (i=0; i<15; i++)

// dst[i] = (src1|src2)ctr[i].sel



AES & RS decoder Performance. Intel® Core™ 2

AES (cycles/byte)

implementation	encode	decode
fast, but unprotected	17	17
protected: V1	50	65
protected: V2	40	40
protected: V3	64	78
protected: Composite Field	28	28

Ability for cache observation

V1: once per round

V2: once per few rounds

V3: multiple times per round

RS decoder (cycles/block)

2GHz Intel® Core™ 2 Duo 32KB/2MB/2GB

stage	IPP (tbl)	IPP (cf)	IPP(cf)	DSP StarCore™ SC140
syndromes	164277	11646	5877	~6000
error locator	5580	2214	2772	~4000
error locations	74187	7416	4464	~4000
error values	10026	1314	729	~600
Total	254070	22590	15174	14600

RS(255,223)

RS(255,239)

Summary

- Considered the Lookup Table, Direct Computation and Composite Fields based implementations of GF(2^8) operations.
- Compared performance of the implementations and made sure that the Composite Field approach is the best on Intel® Architecture as well as on PowerPC and AMD (non existed yet) microprocessors.
- Considered implementation of protected version of AES cipher and RS decoder based on Composite Field approach. Demonstrated their performance advantage over traditional approaches.

