Some properties of intelligence of “smart” structures

A.G. Kolpakov

DAEIMI, Università degli Studi di Cassino, Via Di Biasio, 43 , Cassino (FR) 03043 ITALY

algk@ngs.ru

Abstract. *Analysis of simplest smart structure – smart beam, allows analyze some properties of artificial intellect and its relation to the restrictions on the system.*

Key words: smart structure, intellect, relation to the restrictions on the system

1. THE INTEGRATED DESIGN PROBLEM

The “smart” (called also intelligent, see, e.g., [1]) structure has actuators and control unit among those constitutive elements. Using these devices, the structure can affect themselves and adapt themselves to an external force applied to them, see Fig. 1. In this paper, we consider a simplest model of smart structure, a uniform beam with the clamped ends 0 and 1, subjected to an external force $F(x,t)$ and control forces produced by the actuators. Here t is a parameter used to describe position of the external force. It is required to find the number, positions and switching instruction for the actuators, which minimize deflection $u(x,t)$ of the beam on [0,1].

We pay attention to some properties of the control instruction of the “smart” structure.

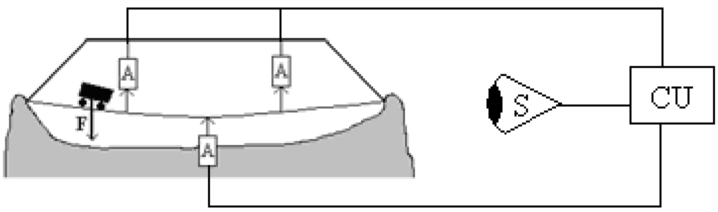

Fig. 1. Intelligent structure: **F -** external force, A – actuators, S – sensors (observer), CU – control unit.

We can describe positions and powers of the actuators by using one function $p(x,t)$ determined as follows: $p(x,t)=0$ if there is no actuator in the point x, $p(x,t)\neq 0$ if an actuator of the power $p(x,t)$ is placed in the point x. Deflection $u(x,t)$ of the beam is determined from solution of the problem

$$u^{IV}= p(x,t)+F(x,t)\ ,\ u(0)=u(1)=0,\ u'(0)=u'(1)=0. \tag{1}$$

Let the powers of actuators be subjected to the following restrictions, see [2] (other types of the restrictions are possible, see [3]),

$$\int_0^1 p(y,t)dy \leq 1,\ \ p(x,t)\geq 0. \tag{2}$$

We have to minimize the deflection $u(x,t)$ of the beam,

$$\|u\|_{C[0,1]} \equiv \max_{x\in[0,1]} |u(x,t)| \to \min \tag{3}$$

with respect to $p(x)$, satisfying (2) with $u(x,t)$ determined from (1).

The condition (3) taken voluntary and it can be chosen for another functional. There existence of such functional is important.

The restrictions (2) do not describe properties of the actuators completely. We consider the pointwice actuators concentrated into points $\{y_1,\ldots,y_m\}$ of possible positions of actuators Then $p(x,t)$ belongs to the functional set

$$\mathrm{A}=\left\{p(x,t): p(x,t)=\sum_{j=1}^{m} p_j(t)\delta(x-y_j)\right\},$$

where $\delta(x)$ is the "delta"–function and $p_j(t)$ has the meaning of the power of the actuator placed at the point y_j (j=1,…,m).

Solutions of the problem (1), can be written in the form [9]

$$u(x,t)= L(x,y)F(y,t)dy+\int_0^1 L(x,y)p(y,t)dy, \tag{4}$$

where $L(x,y)$ is the fundamental solutions of the problem (1).

We choose points $\{x_1,\ldots,x_n\}\subset[0,1]$ (call them the points of observation of deflection). The problem (4) for $x=x_1,\ldots,x_n$ and $p(x)\in\mathrm{A}$ takes the form

$$u(x_i,t)=G(x_i,t)+\sum_{j=1}^{m} L_{ij}p_j(t), \tag{5}$$

where

$$G(x,t)=\int_0^1 L(x,y)F(y,t)dy \tag{6}$$

is the known function and $L_{ij}=L(x_i,y_j)$, $i,j=1,\ldots,m$.

The restrictions (2) for $p(x)\in\mathrm{A}$ take the form

$$\sum_{j=1}^{m} p_j(t)\le 1,\ p(t)_j\ge 0. \tag{7}$$

Introducing vectors $\mathbf{u}(t)=\{u(x_i,t),\ i=1,\ldots,n\}\in R^n$, $\mathbf{y}_j=\{L_{ij},\ i=1,\ldots,n\}\in R^n$, $\mathbf{y}_0(t)=\{\mathrm{G}(x_i,t),\ i=1,\ldots,n\}\in R^n$, we can write (5) as

$$\mathbf{u}(t)= \mathbf{y}_0(t)+\sum_{j=1}^{m} \mathbf{y}_j p_j(t)\in R^n. \tag{8}$$

The sums in (8) under the condition (7) form the cone $P=conv\{\mathbf{y}_1,\ldots,\mathbf{y}_m,0\}$ with the base $P^0=conv\{\mathbf{y}_1,\ldots,\mathbf{y}_m\}$ and the top 0. We note that the cone P does not depend on t. The sums in the right-hand side of (8) under the condition (7) form the "moving" cone $K(t)=P+\mathbf{y}_0(t)$. We note that in (8) only the vector $\mathbf{y}_0(t)$ depends on t.

Discretizing the condition (3), we obtain the following condition

$$\|\mathbf{u}(t)\|\equiv \max_i |u_i(t)|\to\min, \tag{9}$$

where $\mathbf{u}(t)$ is determined by (7), (8). The problem (9) with conditions (7), (8) can be written as

$$\|\mathbf{u}(t)\|\to\min,\ \mathbf{u}\in K(t). \tag{10}$$

One can solve the problem (10) using the methods of the optimal control theory [7] or convex analysis [8]. The authors propose the more effective method is based on the methods similar to the methods of computational geometry.

We consider possible positions of the cone $K(t)$ relatively to the cube $D(c)$ determined by the condition $\|\mathbf{u}\|\le c$. The problem (10) is solvable if there exists $c>0$, such that

$$D(c) \cap K(t) \neq \varnothing. \tag{11}$$

The optimal control corresponds to the minimal c for which (11) is satisfied.
Considering the problem (11), we distinguish two different levels of "intelligence" of the system. They are the high-intelligent system and the system following "if-then" instructions (also known also as knowledge, see, e.g., [10]). The detailed descriptions of these systems are the following.

1. *The high-intelligent system.* The system calculates the powers of actuator for current $t \in T$ by solving the problem $D(c) \cap K(t) \neq \varnothing$. Such kind system can adopt themselves to wide class of external forces.
2. *The system following "if-then" instructions.* The system switch actuators following to the instruction of the form *if* <condition> *then* <switch on/off a grope of actuators> developed in a separate way. Such kind system can word against a specific class of external forces.

The algorithm and examples of developing *"if-then"* instructions can be found in Kolpakov (2003, 2005), Kolpakov&Kolpakov (2006a,b).

2. CONTINUUM PROBLEM

In this section we consider the problem of integrated desing in the case when positions of actuators can be taken arbitrary. In particular, we do not use in this section the notion of possible position of actuators.

In the points of observation

$$\mathbf{u}(t) = \mathbf{y}_0(t) + \int_0^1 \mathbf{L}(y) p(y,t)\, dy, \tag{12}$$

where $\mathbf{u}(t)=\{u(x_i,t),\ i=1,\dots,n\}\in R^n$, $\mathbf{L}(y)=\{L(x_i,y),\ i=1,\dots,n\}\in R^n$, $\mathbf{y}_0(t)=\{G(x_i,t),\ i=1,\dots,n\}\in R^n$.

We take two points of observation: x_1=0.25 and x_2=0.5. Equation (12) with the restrictions (2) describes the cone $K(t)=\mathrm{P}+\mathbf{y}_0(t)$, where $P=conv\{\boldsymbol{\Gamma},0\}$ and $\Gamma = \mathbf{L}(y)=\{\mathrm{L}(0.25,y),\ \mathrm{L}(0.5,y),\ y\in[0,1]\}\in R^2$.
Let the deflecrions $\|\mathbf{u}\|\le c$ in the points of observation be allowed. We construct a system of actuators which guaranty this condition. We explaine the mentod of solution on an example. At Fig. 2 line Γ is shown. It corresponds to the mooving of the pointwise force along the beam between its ends x=0 and x=1. The function $\mathbf{L}(y)$ and the line Γ, were computed numerically. The computations are not complex, but it is difficult carry out them in an explicit form. The line Γ is convex, than $P=conv\{\Gamma,0\}=conv\Gamma$. The square $D(c)$ goes along Γ. Fig. 2 shows the traces of two squares $D(c)$ (on is twice more then another) corresponding to admissible deflections.

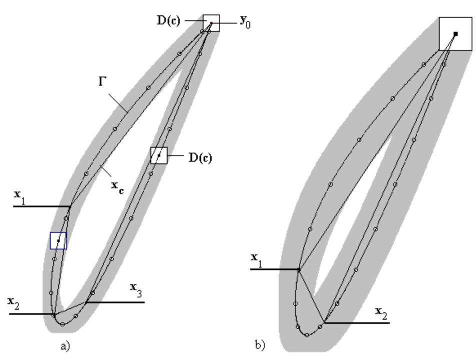

Fig. 2. a) the minimal number of actuators is three, b) the minimal number of actuators is two; the maximal allowed displacements are twice more in the case b) then in the case a).

The possible deflections which can be produced by actruatirs correspond to convex combinations ot the points belonging to the line Γ. Our aim is to find the minimal number of actuatirs, which can keep the deflections in the points of observation in the set $D(c)=\{\mathbf{u} : \|\mathbf{u}\|\le c\}$. It is sufficient find the minimal set of the points $\mathbf{x}_1,\dots, \mathbf{x}_n$, such that the broken line S with the nodes $\mathbf{x}_1,\dots,\mathbf{x}_n$ belongs to the trace of the square $D(c)$. The idea of construction of this line is shown at Fig. 2. At Fig. 2a), the line S has three vertecies $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$ (we do not account the point $\mathbf{y}_0$, which corresponds to the absense of a control). To find positions of the actruators we write the equation of line Γ in the parametric form using the coordinate t along the beam as the parameter. At Fig.2 points on the line Γ correspond to values $t=i/20$ (the lenght of the beam is 1).

For the case shown at Fig. 2a), position of the actuator A_1 is $x_1=0.29$, position of the actuator A_2 is $x_2=0.35$, and position of the actuator A_3 is $x_3=0.58$.

After that we develope the logical part of the progect – the switchign instruction. The switchign instruction is developed using the same Fig. 2a). We express arbitrary point $\mathbf{x}=\mathbf{L}(t_0)$ on the line S in the form of the convex combination $\mathbf{x}= p_1\mathbf{x}_1+p_2\mathbf{x}_2+p_3\mathbf{x}_3$ ($\mathbf{x}=\mathbf{L}(t_0)$). From this representation we obtain the switchign instruction *if* ($t=t_0$) *then* ($p_1(t)=p_1$, $p_2(t)=p_2$, $p_3(t)=p_3$).

At Fig. 2b) solution of the problem is given for the maximal allowed displacements are twice more in the case a). In this case, the number of actuatirs is two.

3. IDENTIFICATION OF POSITION OF APPLICATION OF FORCE

An important element of intelligent structure is an identification system. An identification system consists of sensors (measure units) and a logical unit.

The problem arising in intelligent structure is related to the fact that control hampers the observation. To illustrate this thesis, we present deflections $\mathbf{u}(t) =(u(0.25,t), u(0.5,t))$ of the beam considered in Section 2 when the external unit force $F(x,t)=\delta(x-t)$ moves from the point t=0 to the point t=0.5 and actuators work in optimal regime in accordance with the switching instruction given by Table 1. The deflections ($u(0.25,t)$, $u(0.5,t)$) form a planar curve

$$\Gamma =\{|u(0.25,t)|, |u(0.5,t))| : 0<t<0.5 \ (u(0.25,t), u(0.5,t)) : 0<t<0.5\}. \quad (13)$$

Fig. 3a) displays the curve Γ (13) corresponding to absence of control. Fig. 3b) displays the curve Γ (13) corresponding to optimal control developed in [4].

It is clear that the curve displayed in Fig. 3a) is suitable to solve the problem of identification of position t of the external force $F(x,t)=\delta(x-t)$, when deflections (u_1, u_2) are observed. For this curve, the identification problem is well-posed and stable even against relatively large inaccuracy of measurements of deflections.

For the curve displayed in Fig. 3b), the identification problem is ill-posed. For example, for the deflections $(u_1, u_2)=(0,0)$, the identification problem has three different solutions. In addition, the problem is unstable against relatively large inaccuracy of measurements of deflections. To explain the last thesis, we introduce the circle of measurement error as

$$S=((\delta u_1, \delta u_2) : \delta u_1^2+\delta u_2^2<R^2\}, \quad (14)$$

where δu_1 and δu_2 are errors in measure of u_1 and u_2, correspondingly, R is the measurement error. In Fig. 3, the circles of measurement errors (14) are shown for

$$R=0.05\max\{|u(0.25,t)|, |u(0.5,t))| : 0<t<0.5\}, \quad (15)$$

i.e. for 5% measure error.

It is clear seen that for the displayed in Fig. 3a) and the error R (15), solution of the identification problem can be solved (i. e. position t of the force can be found) with accuracy about 5% in a simple way. Really, anybody can develop computer program solving the identification for the curve displayed in Fig. 3a). Program will be small and fast.

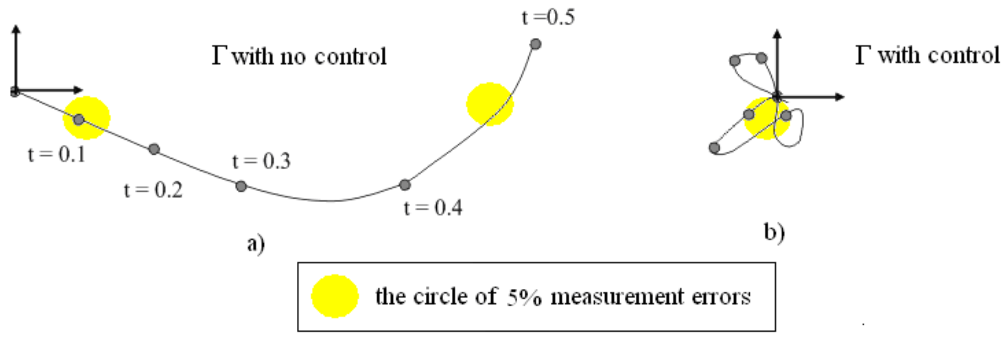

Fig. 3. The deflections $\mathbf{u}(t)=(u(0.25,t), u(0.5,t))$ as the functions of position t ($t \in [0, 0.5]$): a) with no control, b) with optimal control.

For the beam subjected to the optimal control, solution of the identification problem cannot be obtained for the error R (15) with accuracy satisfactory for any practical applications. It is clear seen from Fig. 3b). Since we know switching table and positions of actuators, we can calculate deflection $\mathbf{u}^0(t)$ of beam without control (i.e. curve Fig. 3a)) observing deflection $\mathbf{u}(t)$ of beam without the control (i.e. curve Fig. 3b)) in accordance with the formula $\mathbf{u}^0(t) = \mathbf{u}(t) - \sum_{j=1}^{m} \mathbf{y}_j p_j(t)$. After that, we arrive at the situation corresponding to Fig. 3a) and we can solve the identification problem with no problem.

4. CONCLUSIONS

The main conclusion of this paper is that in "smart" structures the mechanical structure (structural elements + actuators + sensors) and intelligence are strongly related. In the case considered above, the structure (the number and positions of actuators) and intelligence (switching instruction) are determined from the same problem. Similar situation takes place for the identification problem.

We also mention the following general properties of the intelligent structures observed in our works on the subject (this work and papers [4, 5, 6]):

1) A structure demonstrates the existence (or absence) of the intelligence when it (a) has an "aim" (in our consideration this is condition (3)) and (b) is subjected to the action of an inner factor varies.
2) There exist two level of the "intelligence": the high-intelligence and "*if-then*" instructions intelligence (also known is "knowledge"). The "*if-then*" instruction intelligence is suitable for on-the-fly control of a system.
3) The system identifies inner force well (the corresponding problem is well posed) if both reaction and internal control are unknown. Else, the identification problem usually is ill-posed.

The simplicity of our model does not restrict the generality of our conclusions. Our conclusions can be extended to linear systems for which "action–reaction" relationship has a form similar to (7). One can find numerous analogs of the indicated properties in the nature and social systems. In our analysis these properties arise as a result of the mathematical analysis of a simple mechanical object.

We also noted that in the case under consideration the structure and intelligence (knowledge) are developed in optimal way but not by the smart structures itself (the smart structure cannot modify themselves). It makes leads to characterization of "smart" structures as the *homunculus* type object.

5. AKNOWLEDGMENT

The research has received funding from the *European Community's Seventh Framework Programme under grant agreement n° PIIF-GA-2008-219690.*

REFERENCES

[1] Kalamkarov A.L., Kolpakov A.G. *Analysis, design and optimization of composite structures*. John Wiley&Sons. Chichester, New York. 1997

[2] Kolpakov A.G. Design of a Beam with Controllable Force Elements *J. Appl. Mech. and Technical Phys.* 2003, V. 44, N. 4, pp. 584-595,

[3] Kolpakov A.G. The design of a beam with controlled force *units J. Appl. Math. and Mech.* 2005, V. 69, pp. 89-99

[4] Kolpakov A.A. On mathematical modeling of "intelligence". *5th World Congress of Biomechanics.* Munich, Germany, July 29th-August 4th, 2006. Poster Program

[5] Kolpakov A.A., Kolpakov A.G. Design of intelligent beam **-** an integrated design procedure. *Structural and Multidisciplinary Optimization***.** 2006, V.31, N13: 76-80.

[6] Kolpakov A.A., Kolpakov A.G. Design of "intelligent" structures as a discrete optimal problem. *Electronic Notes in Discrete Mathematics,* 2006, 27.

[7] Pontryagin L.S., Boltyanskii V.G., Gamkrelidze R.V., Mishenko E.F. *The Mathematical Theory of Optimal Processes*. Wiley Intersieces. New York. 1962.

[8] Rocafellar R.T. *Convex analysis*. Princeton Univ Press, Princeton, NJ. 1970.

[9] Vladimirov V.S. *Equations of mathematical physics*. Mir Publ., Moscow. 1984.

[10] Zagoruiko N.G. *Applied methods of analysis data and knowledges* Novosibirsk, Inst. Math. Publ. 1999.