

5. Выполненное исследование базируется на развитом алгебро-грамматическом аппарате, включающем логические средства и метаправила выводимости схем алгоритмов и программ, принадлежащих классам, ассоциированным с актуальными предметными областями. Особенность алгебро-грамматических средств представления знаний - гармоническое сочетание декларативных процедурных и трансформационных спецификаций, а также адекватность данных средств концепции объектно-ориентированного программирования.

На базе полученных результатов разработаны наукоемкая технология и ее инструментарий КЕЙС-система МУЛЬТИПРОЦЕССИСТ, нашедшие применение при решении задач АСУ, САПР конструкторской и технологической подготовки производства, языковых процессов транслирующего и интерпретирующего типа.

### *Литература*

1. ГЛУШКОВ В.М., ЦЕЙТЛИН Г.Е., ЮЩЕНКО Е.Л. Методы символьной мультиобработки. - Киев: Наукова думка, 1980. - 252 с.
  2. ДАРЛИНГТОН Дж. Синтез нескольких алгоритмов сортировки //Кибернетический сборник. - №18. - С. 141-176.
  3. ЦЕЙТЛИН Г.Е. Проектирование алгоритмов сортировки: классификация, трансформация, синтез //Программирование. -1989. - №3. - С. 3-24.
- То же //Программирование. - 1989. -№6. -С. 4-19.  
 То же //Кибернетика. - 1989. -№6. -С. 67-74.

### FINITARY LAMBDA CLONES

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There are known several algebraic structures aspiring to be an algebraic counterpart of the  $\lambda$ -calculus:  $\lambda$ -algebras, combinatory models,  $\lambda$ -models (here and further we follow, on the whole, the terminology adopted in [1]). We present one more such a structure called a (finitary)  $\lambda$ -clone which has some attractive features from both universal algebra and  $\lambda$ -calculus points of view. Namely, for any environment model of the  $\lambda$ -calculus, the range of the valuation mapping is a  $\lambda$ -clone, in fact, a so called locally finite (l.f.)  $\lambda$ -clone. In addition, the very valuation mapping is nothing but a homomorphism onto this  $\lambda$ -clone and the corresponding  $\lambda$ -theory (of the model) is the kernel of this homomorphism. Equivalently, if  $(C, ', k, s)$  is a  $\lambda$ -algebra and  $X$  is a countable set of variables, then the polynomial algebra  $C[X]$  can be equipped with the structure of a  $\lambda$ -clone and proves the l.f.  $\lambda$ -clone generated by  $C$  (not  $X$ !) with certain defining relations.

At the same time, the class of all  $\lambda$ -clones is a variety of algebras contained in the class of  $\lambda$ -models and, besides, the last one, considered as a category, is equivalent to the full subcategory of  $\lambda$ -clones consisting of l.f.  $\lambda$ -clones.

These nice properties of  $\lambda$ -clones are provided by the point that variables,  $\lambda$ -quantifiers and substitutions are included directly into the signature of  $\lambda$ -clones. Roughly speaking, a  $\lambda$ -clone is an abstract clone in the sense of universal algebra equipped with operations of application and  $\lambda$ -quantifications.

DEFINITION 1 [2]. Let  $L = (x_i, s_i, ', \lambda_i)_{i < \omega}$  be a signature of operation symbols where all  $x_i$  are constants, all  $s_i$  and  $'$  are binary and all  $\lambda_i$  are unary. A (finitary)  $\lambda$ -clone is defined to be an  $L$ -algebra  $\mathcal{W}$  s.t. the following identities hold for each  $i, j, k < \omega$  with  $i \neq j \neq k$  ( $u, w$  range over the underlying set  $W$ ,  $v$  stands for  $s_j(x_k, w)$ ):

- (S)  $s_i(x_i, w) = w, s_i(w, x_i) = w, s_j(w, x_i) = x_i, s_j(u, v) = v,$   
 $s_i(u, w_1, w_2) = s_i(u, w_1), s_i(u, w_2);$   
 (SQ)  $s_i(w, \lambda_i u) = \lambda_i u, s_i(v, \lambda_j u) = \lambda_j s_i(v, u);$   
 ( $\alpha$ )  $\lambda_i v = \lambda_j s_i(x_j, v);$  ( $\beta$ )  $(\lambda_i w), u = s_i(u, w).$

Here (S) - identities correspond to intuitive understanding of  $s_i(u, w)$  as the result of substituting of  $u$  into  $w$  for  $x_i$ , (SQ) - identities regulate interaction between substitutions and  $\lambda$ -quantification, ( $\alpha$ ), ( $\beta$ ) provide ( $\alpha$ ), ( $\beta$ ) - conversion resp.

Given a  $\lambda$ -clone  $\mathcal{W}$ , for each element  $w \in W$  we introduce its dimension set,  $\Delta w := \{i < \omega: s_i(u, w) \neq w \text{ for some } u \neq x_i\}$ , and call  $w$  finitary if  $|\Delta w| < \omega$  and closed if  $\Delta w = \emptyset$ . Further, we define  $W_{\text{fin}} := \{w: |\Delta w| < \omega\}$  and  $W_\emptyset := \{w: \Delta w = \emptyset\}$ .  $W_{\text{fin}}$  proves a subalgebra of  $\mathcal{W}$ ,  $\mathcal{W}_{\text{fin}}$ , and  $\mathcal{W}$  is called locally finite dimensional (l.f.) if  $\mathcal{W} = \mathcal{W}_{\text{fin}}$ .

DEFINITION 2. An environment domain is defined to be a pentuple  $\mathcal{E} = (D, V, F, \phi, \Psi)$  with  $D$  a set,  $V$  a set of operations  $D^\omega \rightarrow D$ ,  $F$  a set of operations  $D \rightarrow D$ ,  $\phi$  a mapping  $D \rightarrow F$  and  $\Psi$  a mapping  $F \rightarrow D$  s.t. the following conditions are fulfilled (we use the symbol  $\wedge$  for the ordinary set theoretical (meta)

lambda abstraction):

- (E1)  $\pi_i := (\lambda \rho \in D^\omega. \rho_i) \in V$  for all  $i < \omega$ ,  $\bar{d} := (\lambda \rho \in D^\omega. d) \in V$  for all  $d \in D$ ;  
 (E2) if  $u, v \in V$  then  $(\lambda \rho \in D^\omega. v([i/u]\rho)) \in V$  and  $(\lambda \rho \in D^\omega. \phi(u\rho)(v\rho)) \in V$ ;  
 (E3) if  $v \in V$  then, for any fixed  $i < \omega, \rho \in D^\omega$ ,  $(\lambda d \in D. v([i/\bar{d}]\rho)) \in F$ .

An environment domain is said to be an environment model if

- (E4)  $\phi \Psi f = f$  for all  $f \in F$ .

CONSTRUCTION.

(i) With any environment domain  $\mathcal{E}$  as above there is correlated the L-algebra  $\mathcal{V}_{\mathcal{E}} = (V, \pi_i, s_i, ', \lambda_i)$  with operations defined according to the items (E1,2,3) of the definition 2.

(ii) With any L-algebra  $\mathcal{W}$  there is correlated a domain pentuple  $\mathcal{E}_{\mathcal{W}} = (D, V, F, \phi, \Psi)$  where  $D = W_\emptyset$ ,  $V$  is the set of all operations on  $D$  determined by polynomials built from a countable set of variables and elements of  $D$  with the operation  $'$ ,  $\phi w := \lambda u \in W_\emptyset. w'u$ ,  $F := \{\phi w : w \in W_\emptyset\}$ ,  $\Psi \phi w := (\lambda_i \lambda_j. (x_i ' x_j))' w$ .

THEOREM 1. If  $\mathcal{E}$  is an environment model then  $\mathcal{V}_{\mathcal{E}}$  is a l.f.  $\lambda$ -clone; if  $\mathcal{W}$  is a  $\lambda$ -clone then  $\mathcal{E}_{\mathcal{W}}$  is an environment model; finally,  $\mathcal{E}_{\mathcal{W}_{\mathcal{E}}} \cong \mathcal{E}$  and  $\mathcal{V}_{\mathcal{E}_{\mathcal{W}}} \cong \mathcal{W}_{fin}$ .

THEOREM 2. A  $\lambda$ -theory is defined to be a couple  $(C, T)$  with  $C$  a set of constants and  $T$  a subset of  $\Lambda(C) \times \Lambda(C)$  closed under the ordinary  $\lambda$ -calculus deducibility. If  $\mathcal{T} = (C, T)$  is a  $\lambda$ -theory then  $\mathcal{W}_{\mathcal{T}} = \Lambda(C)/T$  is a l.f.  $\lambda$ -clone; if  $\mathcal{W}$  is a  $\lambda$ -clone and  $\mu$  is the homomorphism  $\Lambda(W_\emptyset) \rightarrow \mathcal{W}$  naturally extending the identity inclusion  $W_\emptyset \hookrightarrow W$ , then  $\mathcal{T}_{\mathcal{W}} = (W_\emptyset, \ker \mu)$  is a  $\lambda$ -theory; finally,  $\mathcal{T}_{\mathcal{W}_{\mathcal{T}}} \cong \mathcal{T}$  and  $\mathcal{W}_{\mathcal{T}_{\mathcal{W}}} \cong \mathcal{W}_{fin}$ .

CONJECTURE. The variety generated by the class of all l.f.  $\lambda$ -clones coincides with the class of all  $\lambda$ -clones.

IN PROSPECT. The notion of a finitary  $\lambda$ -clone is somewhat unnatural as finitary operations acting on closed elements can not reach elements with infinite dimension sets. In this con -

text, a more natural structure is a  $\lambda$ -clone with infinitary,  $(1+\bar{i})$ -ry, applications and infinitary,  $\bar{i}$ -ry,  $\lambda$ -quantifiers for all, finite and countable, sequences  $\bar{i} \in \omega^\infty$ . In this way we obtain an algebraic version of an infinitary  $\lambda$ -calculus but this is another story.

## REFERENCES

1. MEYER A.R. What is a model of the lambda calculus? Information and Control, 52,1, 1982(87-122).
2. DISKIN Z.B. Lambda term systems. Z.Math.Logic Grundle Math. Submitted.

## ON CODING OF HEREDITARILY-FINITE SETS, POLYNOMIAL-TIME COMPUTABILITY AND $\Delta$ -EXPRESSIBILITY

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This paper is devoted to computability and definability in terms of bounded (i.e.,  $\Delta$ -) set theoretic language (cf. references below).

A coding (or numbering; cf. the general theory in [3]) of the universe of hereditarily-finite sets HF is any surjection  $\vartheta: A^* \rightarrow HF$  from the set of all finite strings over some finite alphabet A. Let  $P_\vartheta$  denote the class of operations  $F: HF \rightarrow HF$  such that  $F\vartheta = \vartheta f$  for some polynomial-time computable (or shortly, P-) function  $f: A^* \rightarrow A^*$ . For any two codings  $\vartheta: A^* \rightarrow HF$ ,  $\eta: B^* \rightarrow HF$  and P-function  $f: A^* \rightarrow B^*$  the P-reducibility  $\vartheta = \eta f$  is denoted also as  $\vartheta \leq_P^f \eta$  or  $\vartheta \leq_P \eta$ . P-equivalence  $\vartheta \equiv_P \eta$  means  $\vartheta \leq_P \eta$  &  $\eta \leq_P \vartheta$  and implies  $P_\vartheta = P_\eta$ . If cardinalities of A and B are  $\geq 2$  then any  $\vartheta: A^* \rightarrow HF$  is P-equivalent to some  $\vartheta: B^* \rightarrow HF$  (via arbitrary two-sided P-bijection  $f: A^* \rightarrow B^*$ ). Hence, we will usually consider codings over the same A. Any  $\vartheta$  is called P-coding if (1) the predicate " $HF \models \vartheta(a) \in \vartheta(b)$ " is P-decidable on any,  $a, b \in A^*$  and (2) two P-computable mappings  $a \mapsto a_1, \dots, a_k$  and  $a_1, \dots$