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SET FUNCTIONS AND THEIR APPLICATIONS IN THE THEORY OF LEBESGUE AND SOBOLEV SPACES. I

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We study the properties of mappings inducing a bounded operator of Lebesgue or Sobolev spaces by change of variable and the properties of the operator of extension of functions in Sobolev classes beyond the domain of definition. Throughout, we deduce and apply the properties of quasiadditive functions on open subsets of homogeneous spaces. We estimate the integral of the upper derivative of a set function which implies an easy proof of the Lebesgue Integral Differentiation Theorem and existence of a density almost everywhere.

The article consists of two sections. In Section 1, apart from studying the properties of quasiadditive functions, we find necessary and sufficient conditions on a mapping inducing a bounded extension operator in Lebesgue spaces (in Sobolev spaces with weak first-order derivatives).

Key words and phrases: quasiadditive set function, Lebesgue space, Sobolev space, embedding theorems.

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