

**THE ČURGUS CONDITION IN INDEFINITE
STURM–LIOUVILLE PROBLEMS***A. I. Parfenov*

For a finite signed measure μ on $(-1, 1)$ changing its sign at zero, we study the Riesz basis property in the space $L_{2,|\mu|}$ of generalized eigenfunctions of the spectral problem $-u''(x)dx = \lambda u(x)d\mu(x)$, $-1 < x < 1$, $u(-1) = u(1) = 0$. Primarily, our approach is based on the Čurgus criterion. We present a criterion for the basis property in the case of an odd measure and sufficient conditions (in terms of μ) known so far for a measure absolutely continuous with respect to the Lebesgue measure whose support is the whole interval. We prove the Riesz basis property for a degenerate discrete measure of a special form and a new necessary condition for this property. For a dense embedding $V \subset H = H'$ of a reflexive Banach space V into a Hilbert space H and a symmetric unitary (in H) operator J , we consider the interpolation equality $(V, (JV)')_{1/2,2} = H$ applicable to nonlinear evolutionary equations of mixed type. We also exhibit conditions ensuring this equality and generalizing sufficient conditions for the basis property.

Key words and phrases: indefinite spectral problem, Riesz basis, contraction operator, preservation of boundary values, holomorphic functional calculus, the Kato square root problem, mixed type equation.

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