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LOCAL LIMIT THEOREM FOR THE FIRST CROSSING TIME OF A FIXED LEVEL BY A RANDOM WALK

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Let $X, X(1), X(2), \ldots$ be independent identically distributed random variables with mean zero and a finite variance. Put $S(n) = X(1) + \cdots + X(n)$, $n = 1, 2, \ldots$, and define the Markov stopping time $\eta_y = \inf\{n \geq 1 : S(n) \geq y\}$ of the first crossing a level $y \geq 0$ by the random walk $S(n)$, $n = 1, 2, \ldots$. In the case $\mathbb{E}|X|^3 < \infty$, the following relation was obtained in [2]: $\mathbb{P}(\eta_0 = n) = \frac{1}{n\sqrt{n}}\big(R + \nu_n + o(1)\big)$ as $n \to \infty$, where the constant R and the bounded sequence ν_n were calculated in an explicit form. Moreover, there were obtained necessary and sufficient conditions for the limit existence $H(y) := \lim_{n\to\infty} n^{3/2}\mathbb{P}(\eta_y = n)$ for every fixed $y \geq 0$, and there was found a representation for $H(y)$. The present paper was motivated by the following reason. In [2], the authors unfortunately did not cite papers [1, 3] where the abovementioned relations were obtained under weaker restrictions. Namely, it was proved in [1] the existence of the limit $\lim_{n\to\infty} n^{3/2}\mathbb{P}(\eta_y = n)$ for every fixed $y \geq 0$ *under the condition* $\mathbb{E}X^2 < \infty$ *only.* In [3], an explicit form of the limit $\lim_{n\to\infty} n^{3/2}\mathbb{P}(\eta_0 = n)$ was found under *the same condition* $\mathbb{E}X^2 < \infty$ *in the case when the summand* X *has an arithmetic distribution.* In the present paper, we prove that the main assertion in [1] fails and we correct the original proof. It worth noting that this corrected version was formulated in [2] as a conjecture.

Key words and phrases: random walk, first crossing time of a fixed level, arithmetic distribution, nonarithmetic distribution, local limit theorem.

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1. *Eppel M. S.* (1979) A local limit theorem for first passage time, *Sibirsk. Mat. Zh.*, v. 20, N 1, 181–191 (English translation: *Siberian Math. J.*, v. 20, N 1, 130–138 (1979)).

2. *Mogul'skiĭ A. A. and Rogozin B. A.* (2005) A local theorem for the first hitting time of a fixed level by a random walk, *Matem. Tr.*, v. 8, N 1, 43–70 (English translation: *Siberian Adv. Math.*, v. 15, N 3, 1–27 (2005)).

3. *Alili L. and Doney R. A.* (1999) Wiener–Hopf factorization revisited and some applications, *Stochastics Stochastics Rep.*, v. 66, N 1-2, 87–102.