

STURM–LIOUVILLE PROBLEMS IN WEIGHTED SPACES IN DOMAINS WITH NONSMOOTH EDGES. I

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We consider (in general noncoercive) mixed problems in a bounded domain \mathcal{D} in \mathbb{R}^n for a second-order elliptic partial differential operator $A(x, \partial)$. It is assumed that the operator is written in divergent form in \mathcal{D} , the boundary operator $B(x, \partial)$ is the restriction of a linear combination of the function and its derivatives to $\partial\mathcal{D}$ and the boundary of \mathcal{D} is a Lipschitz surface. We separate a closed set $Y \subset \partial\mathcal{D}$ and control the growth of solutions near Y . We prove that the pair (A, B) induces a Fredholm operator L in suitable weighted spaces of Sobolev type, where the weight is a power of the distance to the singular set Y . Finally, we prove the completeness of the root functions associated with L .

The article consists of two parts. The first part published in the present paper, is devoted to exposing the theory of the special weighted Sobolev–Slobodetskiĭ spaces in Lipschitz domains. We obtain theorems on the properties of these spaces; namely, theorems on the interpolation of these spaces, embedding theorems, and theorems about traces. We also study the properties of the weighted spaces defined by some (in general) noncoercive forms.

Key words and phrases: mixed problem, noncoercive boundary condition, elliptic operator, root function, weighted Sobolev space.

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