

SUB-RIEMANNIAN DISTANCE IN THE LIE GROUPS $SU(2)$ AND $SO(3)$

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We calculate distances between arbitrary elements of the Lie groups $SU(2)$ and $SO(3)$ for special left-invariant sub-Riemannian metrics ρ and d . In computing distances for the second metric, we substantially use the fact that the canonical two-sheeted covering epimorphism Ω of $SU(2)$ onto $SO(3)$ is a submetry and a local isometry in the metrics ρ and d . Despite the fact that the proof uses previously known formulas for geodesics starting at the unity, F. Klein's formula for Ω , trigonometric functions, and the conventional differential calculus of functions of one real variable, we focus attention on a careful application of these simple tools in order to avoid the mistakes made in previously published mathematical works in this area.

Key words and phrases: Lie algebra, geodesic, Lie group, invariant sub-Riemannian metric, shortest arc, distance.

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