# ON RENEWAL MATRICES CONNECTED WITH BRANCHING PROCESSES WITH TAILS OF DISTRIBUTIONS OF DIFFERENT ORDERS

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We study irreducible renewal matrices generated by matrices whose rows are proportional to various distribution functions. Such matrices arise in studies of multi-dimensional critical Bellman–Harris branching processes. Proofs of limit theorems for such branching processes are based on asymptotic properties of a chosen family of renewal matrices. In the theory of branching processes, unsolved problems are known that correspond to the case in which the tails of some of the above mentioned distribution functions are integrable, while the other distributions lack this property. We assume that the heaviest tails are regularly varying at the infinity with parameter $-\beta \in [-1, 0)$ and asymptotically proportional, while the other tails are infinitesimal with respect to them. Under a series of additional conditions, we describe asymptotic properties of the first and second order increments for the renewal matrices.

*Key words and phrases*: renewal matrix and its increment, asymptotic representations, regularly varying functions, Bellman–Harris critical processes.

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