SHAPE-PRESERVATION CONDITIONS FOR CUBIC SPLINE INTERPOLATION

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We consider the problem on shape-preserving interpolation by classical cubic splines. Namely, we consider conditions guaranteeing that, for a positive function (or a function whose kth derivative is positive), the cubic spline (respectively, its kth derivative) is positive. We present a survey of known results, completely describe the cases in which boundary conditions are formulated in terms of the first derivative, and obtain similar results for the second derivative. We discuss in detail mathematical methods for obtaining sufficient conditions for shape-preserving interpolation. We also develop such methods, which allows us to obtain general conditions for a spline and its derivative to be positive. We prove that, for a strictly positive function (or a function whose derivative is positive), it is possible to find an interpolant of the same sign as the initial function (respectively, its derivative) by thickening the mesh.

Key words and phrases: cubic spline, shape-preserving interpolation, monotonicity, convexity.

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