

THE GROWTH POINTS OF BOOLEAN VALUED ANALYSIS

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Boolean valued analysis is a powerful method of extending the scope of mathematical theories by means of the special nonstandard models of set theory. This communication will pay attention to the following issues:

1. The Continuum Hypothesis
 - 1.1. Cantor's formulation
 - 1.1.1. The beginning of set theory
 - 1.1.2. The notion of continuum
 - 1.1.3. The Continuum Problem
 - 1.2. Gödel's independence result
 - 1.2.1. The universe of constructible sets
 - 1.2.2. An inner model of $ZFC+CH$ in ZF
 - 1.2.3. The Continuum Hypothesis cannot be disproven
 - 1.3. Cohen's independence result
 - 1.3.1. The universe of Boolean valued sets
 - 1.3.2. A Boolean valued inner model of $ZFC+\neg CH$ in ZFC
 - 1.3.3. The Continuum Hypothesis cannot be proven
 - 1.4. Other independence results
2. Kantorovich spaces
 - 2.1. Key notions
 - 2.1.1. The notion of vector lattice
 - 2.1.2. The notion of Kantorovich space
 - 2.1.3. The Boolean algebra of band projections
 - 2.2. Boolean valued interpretation of the reals
 - 2.2.1. Kantorovich's heuristic transfer principle
 - 2.2.2. Gordon's theorem
 - 2.2.3. The birth of Boolean valued analysis
3. The machinery of Boolean valued analysis
 - 3.1. Ascents and descents
 - 3.2. The Boolean valued transfer
 - 3.3. Applications
 - 3.3.1. Intrinsic characterization of subdifferentials
 - 3.3.2. General desintegration in Kantorovich space
 - 3.3.3. The Kaplansky Problem: Homogeneity of a type I AW^* -algebra
 - 3.3.4. The Wickstead Problem: Order boundedness of band preserving operators
 - 3.3.5. The Maharam extension of a positive operator
 - 3.3.6. Classification of injective Banach lattices

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