or their derivatives as data are more adequate than everywhere defined functions. The corresponding correction of the inverse problem is realized by the composition of the problem Q^{-1} and the auxiliary problem R with domain of data $\operatorname{Dom} R = (\mathbb{R}^3)^3$, domain of unknowns $\operatorname{Im} R = C^1(\mathbb{R})^2$, and condition

$$R\big((t,\alpha,\beta),(x,y)\big) \;\Leftrightarrow\; \begin{cases} x(t_1)=\alpha_1,\ x(t_2)=\alpha_2,\ x(t_3)=\alpha_3, \\ \dot x(t_1)=\beta_1,\ \dot x(t_2)=\beta_2,\ \dot x(t_3)=\beta_3, \end{cases}$$

where $t,\alpha,\beta \in \mathbb{R}^3$, $x,y \in C^1(\mathbb{R})$.

Theorem 1. *If $t,\alpha \in \mathbb{R}^3$ meet the condition*

$$\Delta := \begin{vmatrix} 1 & \alpha_1 & h(\alpha_1,t_1) \\ 1 & \alpha_2 & h(\alpha_2,t_2) \\ 1 & \alpha_3 & h(\alpha_3,t_3) \end{vmatrix} \neq 0 \tag{1}$$

then, given arbitrary $\beta \in \mathbb{R}^3$, the problem $Q^{-1}\circ R$ is uniquely solvable for the data (t,α,β), and its solution $(f_1,f_2,f_3) = (Q^{-1}\circ R)^{\mathrm{s}}(t,\alpha,\beta)$ can be calculated by Cramer's formulas $f_i = \Delta_i/\Delta$, where Δ_i is the determinant of the matrix formed from the matrix in (1) *by replacing the i th column by $\beta = (\beta_1,\beta_2,\beta_3)$.*

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Nonclosed Archimedean cones

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A *wedge* is a nonempty convex subset K of a vector space (henceforth, vector spaces are assumed to be defined over the field of reals) which meets the condition $(\forall\,\alpha \geqslant 0)(\alpha K \subseteq K)$. A wedge K is a *cone* whenever $K \cap (-K) = \{0\}$. As is known, in every (pre)ordered vector space $(X,\leqslant)$ the set $\{x \in X : x \geqslant 0\}$ is a cone (wedge). Conversely, if a cone (wedge) K is fixed in a vector space X

then the order $\leqslant_K$ defined by the rule $x \leqslant_K y \Leftrightarrow y - x \in K$ makes X into a (pre)ordered vector space.

A (pre)ordered vector space $(X, \leqslant)$ is *Archimedean* if for all $x, y \in X$ $(y \geqslant 0)$ the condition $(\forall\, n \in \mathbb{N})(x \leqslant \frac{1}{n}y)$ implies $x \leqslant 0$. A cone (wedge) K in a vector space X is *Archimedean* if so is the (pre)ordered space $(X, \leqslant_K)$. Generalize the notion of Archimedean wedge as follows: a convex set $C \subseteq X$ is *Archimedean* whenever for all $x, y \in X$ the condition $(\forall\, n \in \mathbb{N})(x + \frac{1}{n}y \in C)$ implies $x \in C$. It is easy to see that, for wedges, the latter is equivalent to the above definition.

The main information on Archimedean cones can be found in [1].

The following proposition clarifies the notion of Archimedean set.

Proposition 1. *Given a convex set C in a vector space X, the following are equivalent:*

(a) *C is Archimedean;*

(b) *for all $x, y \in X$ the condition $(\exists\, \varepsilon > 0)(x +]0, \varepsilon]y \subseteq C)$ implies $x \in C$;*

(c) *the complement $X \setminus C$ coincides with its algebraic interior;*

(d) *the intersection of C with every line is closed;*

(e) *the intersection of C with every finite-dimensional subspace of X is closed.*

Note that every convex set which is sequentially closed in any vector topology is obviously Archimedean. In order to give another description of Archimedean sets, we introduce an auxiliary notion: A topological vector space is *sequentially total* if all linear functionals on it are sequentially continuous.

Lemma 1. *Let X be a sequentially total space and let $C \subseteq X$ be a convex set. Then C is Archimedean if and only if C is sequentially closed.*

The problem is of interest of describing the topological vector spaces which include nonclosed Archimedean cones. In what follows, we present the main results obtained on the way to solving this problem as well as its variations (in the statement, cones can be replaced by wedges, while closedness by sequential closedness). It is immediate that, in finite-dimensional spaces, all Archimedean wedges (moreover, all Archimedean convex sets) are closed.

A criterion is rather easily obtained for existence of Archimedean cones or wedges which are not sequentially closed.

Lemma 2. *Given a topological vector space X, the following are equivalent:*

(a) *X is sequentially total;*

(b) *every Archimedean wedge in X is sequentially closed;*

(c) *every Archimedean cone in X is sequentially closed.*

The main problem is solved for a wide class of topological vector spaces of uncountable dimension.

Theorem 1. *Every locally convex space of uncountable dimension includes a nonclosed Archimedean cone.*

For spaces of countable dimension, we currently succeeded only in obtaining a criterion for existence of a nonclosed Archimedean wedge.

Theorem 2. *A topological vector space X of countable dimension includes a nonclosed Archimedean wedge if and only if there is a discontinuous linear functional on X.*

The following theorems specify the boundaries of the class of spaces of countable dimension which include nonclosed Archimedean cones.

Theorem 3. *If a topological vector space contains a nonclosed linearly independent set, then it contains a nonclosed Archimedean cone.*

Theorem 4. *If X is a topological vector space of countable dimension and all linear functionals on X are continuous, then all Archimedean convex sets in X are closed.*

Hypothesis 1. A topological vector space X of countable dimension includes a nonclosed Archimedean cone if and only if there is a discontinuous linear functional on X.

Hypothesis 2. A topological vector space X of countable dimension includes a nonclosed Archimedean cone if and only if X includes a nonclosed linearly independent set.

The above hypotheses are not equivalent: there are examples of topological vector spaces of countable dimension in which all linearly independent sets are closed, while not all linear functionals are continuous.

The main results of the research are published in [2].

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