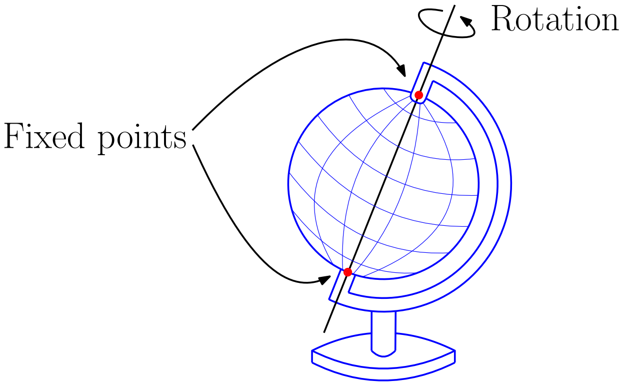
You suck with mixing your coffee and this is why

Suppose we have a globe. This is a (world)ball with the North and South Poles attached to a stand. We can spin it around, which will move every point on the surface of the ball to another point on the surface. Two points, however, will not change positions: the poles. These points are called the fixed points of the rotation.



Fixed point theory studies the existence and properties of fixed points for transformations and deformations of different objects. This theory appears in all areas of mathematics (without too much modesty) and has far-ranging consequences in plenty of scientific disciplines.

One of the most valuable results in fixed point theory is Brouwer's Theorem, named after Dutch mathematician Luitzen Egbertus Jan Brouwer. It says that if a transformation is continuous, i.e. points that are close to each other will, after being transformed, still be close to each other, then this transformation has at least one fixed point.

This theorem has a multitude of applications. Suppose you have filled your mug with KU Leuven logo (this is important!) with coffee, and you mix it with a spoon. By mixing, you move the molecules of the coffee from one place to another. This movement is continuous: if two molecules are initially close to each other, then after mixing they will still be close to each other. Brouwer's Theorem states that this mixing has a fixed point. So, there is a molecule of the coffee that, after mixing, will be at the exact same position as it was before. Therefore, however sad it may be, even if you keep mixing your coffee all day, you can never mix it completely.

Another interesting situation is as follows: suppose you are walking around, holding a map of Belgium in your hands. Then the Belgium on your map is the result of a continuous transformation (namely, resizing) of the actual surface of Belgium. Brouwer's Theorem then asserts there is a point on your map that is exactly above the corresponding point in Belgium. In fact, this is still true if you flip your map over, fold it repeatedly, or even crumple it into a ball – just don't tear it to pieces!

A third and final consequence is what mathematicians lovingly refer to as the "hairy ball theorem". This theorem is often phrased as "it's impossible to comb the hair on a coconot without creating a cowlick". A more useful interpretation, perhaps, is that there is always at least one place on Earth that is perfectly windless.

These examples show that a single theorem in fixed point theory already has plenty of unexpected consequences. But fixed point theory is not just very interesting within mathematics itself, it also has an enormous number of applications in fields like physics, economics and biology. Here, one is interested in the behaviour of a particular system, and especially in its equilibrium states. To study this behaviour, one models the system mathematically and studies the fixed points, which often correspond to the equilibria of the system.

A well-known example is the Nash equilibrium. In a non-cooperative game, this equilibrium is attained when no player benefits from unilaterally changing their strategy. John Nash, known from the movie "A Beautiful Mind", used a generalisation of Brouwer's Theorem to show that every non-cooperative game admits a Nash equilibrium. This result, among others, earned him the Nobel Prize in Economics in 1994.

A lot of mathematicians around the world are working on different topics concerning fixed point theory, some of them working from a small campus in Kortrijk and they wrote this text specially for you.