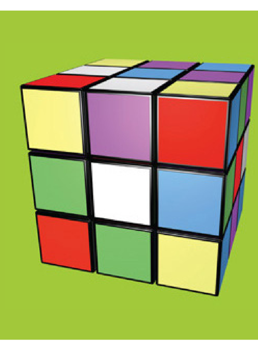

Faculty of Science

Linear groups and their symmetries

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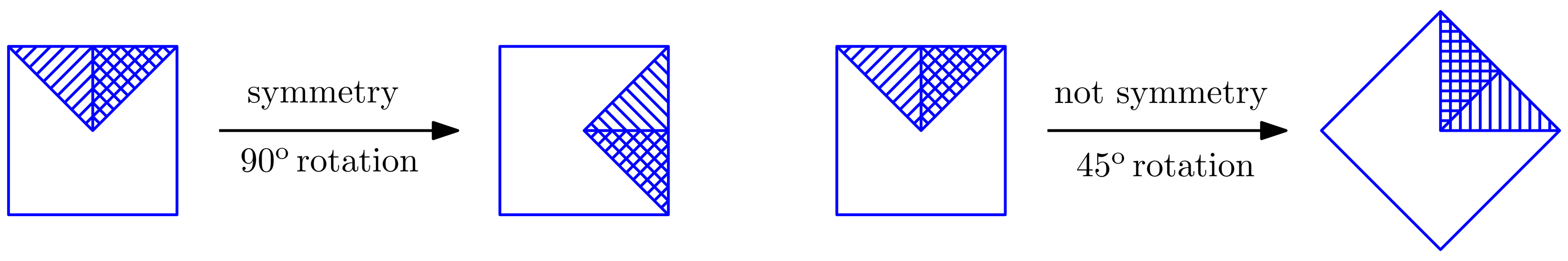
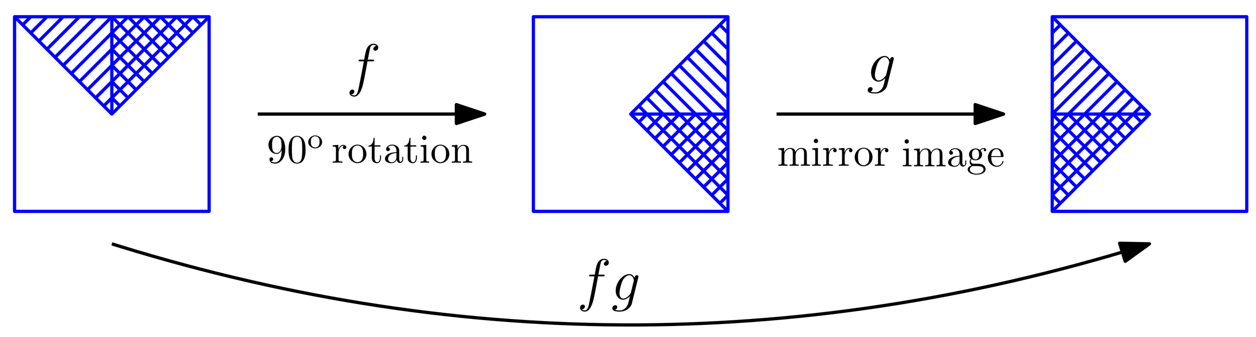
What is group theory

Let us take a geometric figure and move it. If in the end the figure has the same place, then such motion is called a symmetry. The example for a square is below.



The square has 8 symmetries, but the circle has infinitely many. That's why the circle seems more symmetric then the square.
If we first apply a symmetry f and then a symmetry g, we obtain a symmetry fg which is called a product of f and g. For example, on the figure below we clockwise rotate the square for 90^o (symmetry f) and then reflect it with respect to vertical axis (symmetry g).

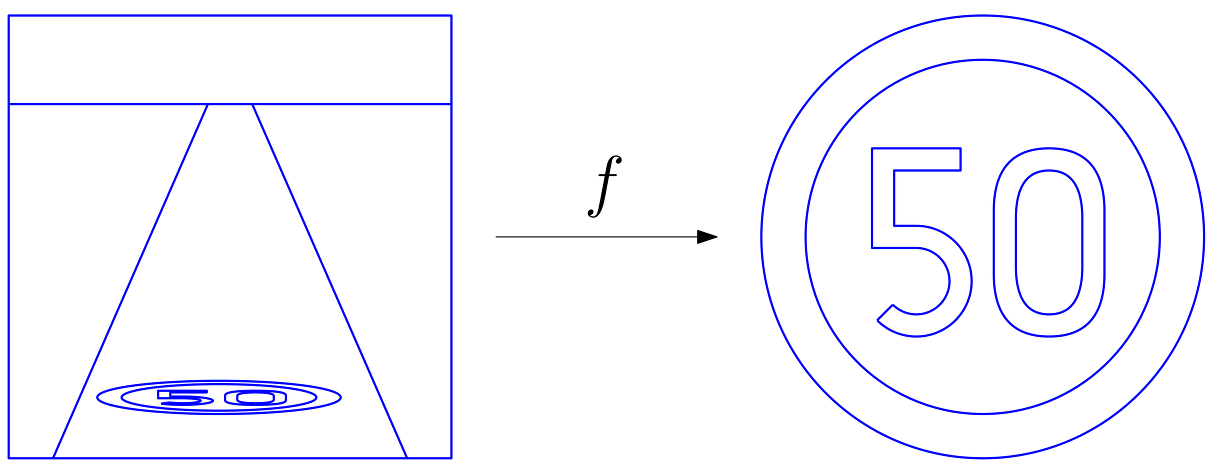


The set of symmetries and all their possible products for a given object is called a group. Group theory studies properties of groups.

The main idea #1: Group theory studies symmetries of different objects (for example, of geometric figures).

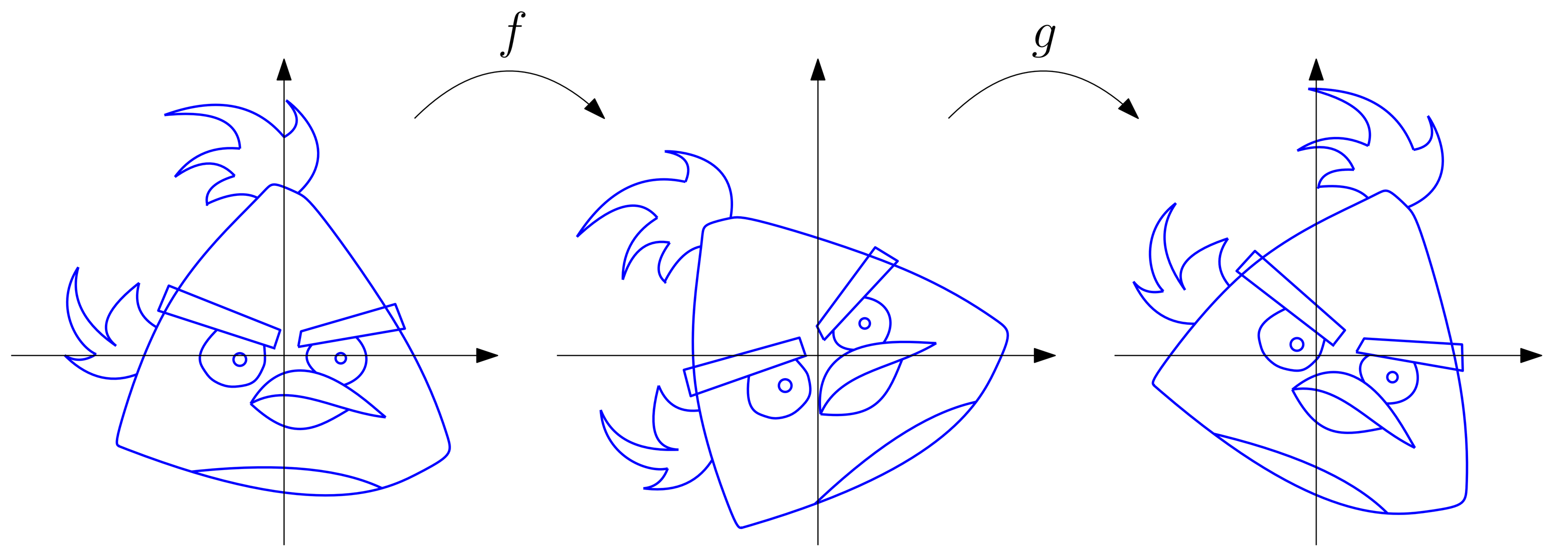
What are linear groups

An important class of groups are linear groups: they describe symmetries of space. This symmetries deform objects in the space, what gives useful applications.
Example 1: Diagonal linear group $\mathrm{D}_n(\mathbb{R})$ describes compressions and extensions of space along the axis. It is used for image recognition. For example, for understanding by computer which road sign is depicted on the road.



Expanding of the picture is realised in computer using diagonal group.

Example 2: Orthogonal linear group $\mathrm{O}_n(\mathbb{R})$ describes symmetries of space which do not change the size and the form of objects. This group is used for simulation of movements: every movement in game "Angry Birds" is programmed using orthogonal groups.



The main idea #2: It is important to study groups since they have a lot of useful applications.

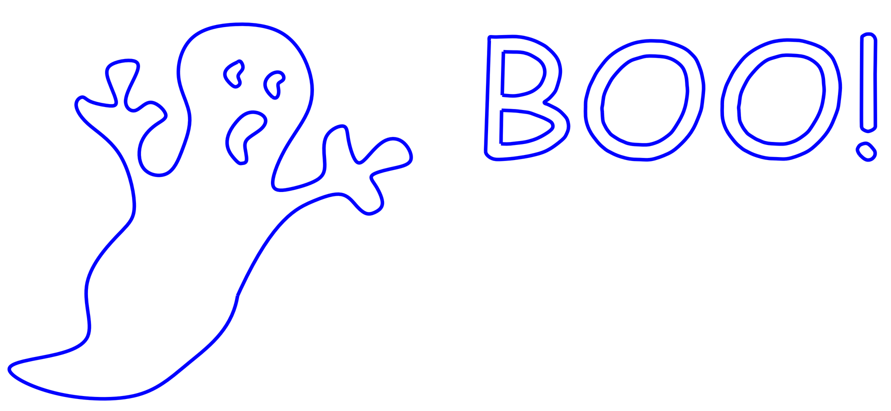
Closer to my research

Groups also have symmetries: the map φ from a group G to itself is called a symmetry of G if for all f, g from G we have $\varphi(fg) = \varphi(f)\varphi(g)$.
I study symmetries of linear groups and their properties. It has applications, for example, in geometrical Nielsen fixed point theory.

The main idea #3: I study properties of symmetries of linear groups with applications in geometry.

Finally, I would like to scare You by my new wonderful result.

Theorem (T. Nasybullov, A. Fel'shtyn, 2016): Let $\mathbb{F}$ be an algebraically closed field such that the transcendence degree of $\mathbb{F}$ over $\mathbb{Q}$ is finite. If a reductive linear algebraic group G over the field $\mathbb{F}$ has a nontrivial quotient group $G/R(G)$, where $R(G)$ is the radical of G, then G possesses the R_∞-property.



Danger: Please do not try to understand this theorem if You are not an expert in group theory.

The theorem above is written to demonstrate the following idea.

The main idea #4: Group theory is not just simple pictures. It is difficult subject which requires deep investigation.

Remark: It is certainly easy to explain physical or biological problem since there are a lot of visual materials in this fields.

Mathematics is more abstract and it requires more efforts and concentration for understanding.

So, I suggest You to read the poster again and comprehend the main ideas one more time.