Generalized Computable Models and Montague Semantics

Artem Burnistov and Alexey Stukachev

Abstract We consider algorithmic properties of mathematical models which are used in computational linguistics to formalize and represent the semantics of natural language sentences. For example, finite-order functionals play a crucial role in Montague intensional logics and formal semantics for natural languages. We discuss some computable models for the spaces of finite-order functionals based on the Ershov-Scott theory of domains and approximation spaces. As another example, in the analysis of temporal aspects of verbs the scale of time is usually identified with the ordered set of real numbers or just a dense linear order. There are many results in generalized computability about such structures, and some of them can be applied in this analysis.

Key words: Montague semantics, intensional logic, functionals of finite types, generalized computability, Σ-predicates

1 Introduction

This paper continues the work started in [17, 16, 3] on algorithmic issues of formal semantics for natural languages. R. Montague in [10, 11] proposed a model-theoretic approach to semantics of English known as Montague intensional logic. It is a typed

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higher-order logic which uses finite types and finite-order functionals to formalize grammar categories of natural languages (in particular, English).

Neither Montague nor other researchers (to our knowledge) studied complexity issues and algorithmic aspects of objects and constructions of this theory. A natural question is to construct computable or effective (in some sense) presentations of rather complicated structures considered in intensional logic. Such presentations allow to regard the meaning of a natural language sentence as an algorithm of checking its relevance (truth value) to a given circumstances (knowledge base).

Our approach is based on the framework of generalized computability formalised via Σ-definability in admissible sets or superstructures developed by J. Barwise [1], Y. N. Moschovakis [12], Yu. L. Ershov [8], and also by R. Montague [9]. To handle finite-order functionals in an effective way we use Ershov-Scott theory of approximation spaces and domains, see [14, 6, 7, 8]. As a useful benefit, such approach allows to consider the case when the basic entities can be approached only via approximations. Also, it becomes possible to study spaces of truth values more complicated than just 0 (“no”) and 1 (“yes”).

Intensionality in models discussed in this paper is limited only to the scale of time as the set of possible worlds, we do not consider modality issues for simplicity reasons. Time in linguistics is usually represented by the ordered set of real numbers (denoted here as $\mathbb{R}$). This model is sufficient to describe formally (and hence analyse effectively) such important features of verbs as tense and aspect, see [2]. There are some properties of dense linear orders (e.g., elimination of quantifiers and decidability), which are well-known for logicians and which could be useful in the analysis of algorithmic properties of interval semantics for verbs in natural languages. The examples of results of this kind can be found in [17, 16].

In this paper, we consider two different kinds of models of entity spaces, namely, rank models and vector models. Vector models are more natural from the approximation-space point of view, and rank models are more natural from the set-theoretical point of view. We prove that type hierarchies based on these classes of models are effectively equivalent. This result is obtained for two different spaces of truth values.

The authors are grateful for anonymous referees for valuable remarks and suggestions. A comparison of our approach with the existing ones will be discussed in a series of forthcoming publications. Here we just present examples of computable models of finite-order functionals and approximation spaces not described before and relevant to linguistics. We believe our approach is closely connected to the problem of understanding the meaning of a natural language sentence via rigorous mathematical formalization. In particular, since we consider effective (in some general sense) models of Montague intensional logic, we plan to describe how exactly our research is connected to the approach by Y. N. Moschovakis named “meaning as an algorithm” [13].

2 Basic Notions of Generalized Computability

Hereditary finite superstructures are the "simplest" examples of models of theory KPU proposed by S.Kripke, R.Platek, J.Barwise, and Yu.L.Ershov for studying generalized computability via Σ-definability in admissible sets (see [1, 8, 15]).

By ω we denote the set of natural numbers. For arbitrary set M, we construct the set $HF(M)$ of hereditarily finite sets over M as follows:

$HF_0(M) = \varnothing$
$HF_{n+1}(M) = \mathcal{P}_\omega(M \cup HF_n(M)),\ n < \omega$
(here $\mathcal{P}_\omega(X)$ is the set of all finite subsets of X)
$HF(M) = \bigcup\limits_{n<\omega} HF_n(M)$

If $\mathfrak{M}$ is a structure of some relational signature σ then one can define on $M \cup HF(M)$ a structure $\mathbb{HF}(\mathfrak{M})$ of signature $\sigma' = \sigma \cup \{U, \varnothing, \in\}$ ($U, \varnothing$, and $\in$ are some symbols not in σ) with the following interpretation of signature symbols:

$U^{\mathbb{HF}(\mathfrak{M})} = M$
$P^{\mathbb{HF}(\mathfrak{M})} = P^{\mathfrak{M}},\ P \in \sigma$
$\varnothing^{\mathbb{HF}(\mathfrak{M})} = \varnothing \in HF_0(M)$
$\in^{\mathbb{HF}(\mathfrak{M})} = \in \cap\, ((M \cup HF(M)) \times HF(M))$

A class of Δ_0-*formulas* of signature σ' is the least one containing atomic formulas which is closed under $\vee$, $\wedge$, $\rightarrow$, $\neg$, and bounded quantifiers $\forall x \in y$ and $\exists x \in y$ ($\forall x \in y\, \varphi$ and $\exists x \in y\, \varphi$ are abbreviations for $\forall x(x \in y \rightarrow \varphi)$ and $\exists x(x \in y \wedge \varphi)$, respectively).

A class of Σ-*formulas* of signature σ' is the least one containing Δ_0-formulas and closed under $\vee$, $\wedge$, bounded quantifiers $\forall x \in y$, $\exists x \in y$, and $\exists x$. As usual, a set is called Σ-definable if it is definable by some Σ-formula with parameters, and Δ-definable if it and its complement are Σ-definable.

3 Montague Intensional Logic

Let e, t, and s be the some fixed symbols used, correspondingly, as names for basic types of entities and truth values, and for marking an intensional shift, i.e., relativization to a state or situation.

Definition The set $Types_{IL}$ is defined as follows:

- $t \in Types_{IL}$, $e \in Types_{IL}$
- if $a \in Types_{IL}$ and $b \in Types_{IL}$ then $(a \rightarrow b) \in Types_{IL}$
- if $a \in Types_{IL}$ then $(s \rightarrow a) \in Types_{IL}$ □

The language of intensional logic IL (see [4, 5, 10, 11]) contains countably many constants of any type $a \in Types_{IL}$ and countably many variables of each type $a \in Types_{IL}$.

A model of intensional logic IL is a quadruple $\langle A, W, T, \leq, F\rangle$ such that A, W, T are nonempty sets, $\leq$ is a linear order on T, F is a function defined on the set of constants of IL as described below. Sets W and T correspond to the sets of possible worlds and time moments correspondingly.

Definition The set D_τ of possible denotations of type $\tau \in Types_{IL}$ is defined by induction on complexity of τ:

- $D_e = A$, $D_t = \{0, 1\}$
- $D_{(a\to b)} = D_b^{D_a}$ (the set of functions from D_a to D_b)
- $D_{(s\to a)} = D_a^{W\times T}$ (the set of functions from $W \times T$ to D_a) □

We denote by S_a the set $D_{(s\to a)}$. Function F defines for each constant of type a some element from S_a which is called its *intension*. Elements from D_a are called *extensions* of type a.

Finite types are used to represent grammar categories (parts of speech) of natural languages. Some correspondences between categories and types are listed in Table 1.

Table 1 Categories and Types of Some Expressions

Category	Grammar equivalent	Corresponding type	Basic expressions
e	no	e	no
t	sentences	t	no
IV	intransitive verbs	(e → t)	walk, talk
CN	common nouns	(e → t)	man, woman
TV	extensional transitive verbs	(e → (e → t))	love, find
CN/CN	extensional adjectives	((e → t) → (e → t))	tall, young
CN/CN	extensional adverbs	((e → t) → (e → t))	rapidly, slowly
T	noun phrases and proper names	((s → (e → t)) → t)	John, ninety, he
t/t	sentence determinants	((s → t) → t)	necessarily, possibly
IV/t	connective verbs	((s → t) → (e → t))	believe, assert

For example, proper names correspond to the type $((s \to (e \to t)) \to t)$ – the set of properties true for the individual with this name. Here we do not consider one of the most complex cases, intensional transitive verbs with the type $((s \to ((s \to (e \to t)) \to t)) \to (e \to t)))$.

Extension (the set of denotations) of type a is the set of possible values of the grammar category interpreted by type a in a model of intensional logic. Correspondingly, intension of type a is a function from $W \times T$ to the extension of type a.

4 Ershov-Scott Functional Spaces

To construct an effective model of Montague intensional logic we apply the domain theory proposed by D. S. Scott [14] and the theory of functional spaces of finite types proposed by Yu. L. Ershov [6, 7, 8]. The definitions below are from [8].

Let $\mathbb{A}$ be a model of KPU (see [8]). If $a \in A$ then $p_l^* a = \{b \mid \exists c(\langle b, c\rangle \in a)\}$, $p_r^* a = \{b \mid \exists c(\langle c, b\rangle \in a)\}$. If $B \subseteq A$ then $B^* = \{b \mid b \subseteq B$ and $b \in A\}$.

The notion of effectively presented functional space is based on the general

Definition Quadruple $\mathcal{B} = \langle B, \leq, Cons, \sqcup\rangle$ is called an *f-base on* $\mathbb{A}$ (see [6, 7]) if the following holds:

1) B is a Δ-definable subset of $\mathbb{A}$
2) $\leq$ is a Δ-definable preorder on B;
let $[B]$ be the quotient of set B by the equivalence relation $\equiv$ defined by the preorder $\leq$ ($b_0 \equiv b_1 \Leftrightarrow b_0 \leq b_1$ and $b_1 \leq b_0$); as usual, $[b]$ denotes the element of $[B]$ which is the equivalence class of $b \in B$; if $C \subseteq B$ then $[C] = \{[b] \mid b \in C\}$; we also use $\leq$ to denote the order induced on $[B]$ by the original preorder $\leq$
3) $Cons$ is a Δ-definable subset of $B^* \setminus \{\emptyset\}$, and for any $b_* \in B^*$ holds

$$b_* \in Cons \Leftrightarrow (\exists b \in B)(\forall b' \in b_*)(b' \leq b)$$

4) $\sqcup : Cons \to B$ is a Σ-definable function such that $[\sqcup b_*]$ for any $b_* \in Cons$ is the least upper bound of $[b_*] \subseteq [B]$ in $\langle [B], \leq\rangle$ □

Definition Let $\mathcal{B}_0 = \langle B_0, \leq_0, Cons_0, \sqcup_0\rangle$ and $\mathcal{B}_1 = \langle B_1, \leq_1, Cons_1, \sqcup_1\rangle$ be some f-bases on $\mathbb{A}$. A direct product $\mathcal{B}_1 \times \mathcal{B}_2$ of $\mathcal{B}_0$ and $\mathcal{B}_1$ is the f-base $\langle B_0 \times B_1, \leq, Cons, \sqcup\rangle$, where $\leq$, $Cons$ and $\sqcup$ are defined as follows:

1) $\langle b_0, b_1\rangle \leq \langle b_0', b_1'\rangle$ iff $b_0 \leq_0 b_0'$ and $b_1 \leq_1 b_1'$ for every $b_0, b_0' \in B_0$ and every $b_1, b_1' \in B_1$
2) $b_* \in Cons$ iff $p_l^*(b_*) \in Cons_0$ and $p_r^*(b_*) \in Cons_1$ for every $b_* \in (B_0 \times B_1)^*$
3) $\sqcup b_* \leftrightharpoons \langle \sqcup_0 p_l^*(b_*), \sqcup_1 p_r^*(b_*)\rangle$ for every $b_* \in Cons$ □

In case the set $Cons$ of mutually consistent fragments (approximations) should be as large as possible, we need

Definition Quadruple $\mathcal{B} = \langle B, b_0, \leq, \sqcup\rangle$ is called an *f^*-base on* $\mathbb{A}$ if $\langle B, \leq, B^* \setminus \{\emptyset\}, \sqcup\rangle$ is an f-base on $\mathbb{A}$, $[b_0]$ is the least element in $\langle [B], \leq\rangle$ and $\sqcup\emptyset = b_0$. □

In general, the range (the set of possible values) of a functional can be arbitrary, so the notion of f^*-base is at hand in the following

Definition Let $\mathcal{B}_0 = \langle B_0, \leq_0, Cons_0, \sqcup_0\rangle$ be an f-base, $\mathcal{B}_1 = \langle B_1, b_1, \leq_1, \sqcup_1\rangle$ be an f^*-base. A functional product $F(\mathcal{B}_0, \mathcal{B}_1)$ of f-base $\mathcal{B}_0$ and f^*-base $\mathcal{B}_1$ is the f^*-base $\langle (B_0 \times B_1)^*, \emptyset, \leq, \sqcup\rangle$, where $\leq$ and $\sqcup$ are defined as follows:

1) $f_0 \leq f_1$ iff $\forall b_0 \in p_l^* f_0(\sqcup_1\{b_1 \mid \exists b_0' \in p_l^* f_0(b_0' \leq_0 b_0$ and $\langle b_0', b_1\rangle \in f_0)\} \leq_1$
$\leq_1 \sqcup_1\{b_1 \mid \exists b_0' \in p_l' f_1(b_0' \leq_0 b_0$ and $\langle b_0', b_1\rangle \in f_1)\})$ for $f_0, f_1 \in (B_0 \times B_1)^*$
2) $\sqcup f_* \leftrightharpoons \cup f_*$ for every $f_* \in ((B_0 \times B_1)^*)^*$ □

Definition For an f-base $\mathcal{B} = \langle B, \leq, Cons, \sqcup\rangle$, the family $I_\Sigma(\mathcal{B})$ of Σ-ideals in $\mathcal{B}$ consists of nonempty Σ-definable subsets $C \subseteq B$ such that

1) from $c \in C, b \in B, b \leq c$ it follows that $b \in C$
2) from $c \in C^*$ it follows that $c \in Cons$ and $\sqcup c \in C$ □

We define a topology on the set $I_\Sigma(\mathcal{B})$ by fixing the basis

$$V_b \leftrightharpoons \{C \mid C \in I_\Sigma(\mathcal{B}), b \in C\}, b \in B.$$

The set $I_\Sigma(\mathcal{B})$ together with the topology specified above is called the *space of* Σ*-ideals* of f-base $\mathcal{B}$. The space $I_\Sigma(\mathcal{B})$ is a topological T_0-space.

Let $\mathcal{B}_0$ be an f-base and let $\mathcal{B}_1$ be an f^*-base. For any ideal I of f-base $F(\mathcal{B}_0, \mathcal{B}_1)$ we can define the continuous function $f_I : I_\Sigma(\mathcal{B}_0) \to I_\Sigma(\mathcal{B}_1)$ as follows. Let $I_0 \in I_\Sigma(\mathcal{B}_0)$. We define

$$f_I(I_0) \leftrightharpoons \{b_1 \mid b_1 \in B_1, (\exists c^* \in I)(\exists b_0 \in I_0)\exists b_1^{'}(b_1 \leq_1 b_1^{'} \text{ and } \langle b_0, b_1^{'}\rangle \in c^*)\}.$$

If $\{\langle b_0, b_1\rangle\} \in I, b_0 \in I_0$, then $b_1 \in f_I(I_0)$.

The mapping $I \to f_I$ from $I_\Sigma(F(\mathcal{B}_0, \mathcal{B}_1))$ to $C(I_\Sigma(\mathcal{B}_0), I_\Sigma(\mathcal{B}_1))$ (the set of all continuous functions from the space $I_\Sigma(\mathcal{B}_0)$ to the space $I_\Sigma(\mathcal{B}_1)$) is injective.

To introduce the simplest example of spaces for entities and truth values, let $\mathcal{A} = \langle A, =, P_1(A), \cup\rangle$, where $P_1(A) \leftrightharpoons \{\{a\} \mid a \in A\}$. This quadruple is an f-base with $I_\Sigma(\mathcal{A}) = P_1(A)$. Also, let α be an arbitrary ordinal in $\mathbb{A}$ and let $\mathcal{B}_\alpha = \langle \alpha, \emptyset, \subseteq, \cup\rangle$. This quadruple is an f^*-base with $I_\Sigma(\mathcal{B}_\alpha) = (\alpha + 1) \setminus \emptyset$. Further on we consider the case $\alpha = 2$.

Definition The set of functional types $Types_f$ together with its proper subset $PTypes_f$ are defined as follows:

1) $\boldsymbol{o} \in Types_f \setminus PTypes_f, B \in PTypes_f \subseteq Types_f$;
2) if $\tau_0, \tau_1 \in Types_f(PTypes_f)$ then $(\tau_0 \times \tau_1) \in Types_f(PTypes_f)$;
3) if $\tau_0 \in Types_f, \tau_1 \in PTypes_f$ then $(\tau_0 \to \tau_1) \in PTypes_f$. □

Definition For every type $\tau \in Types_f$, the f-base $\mathcal{F}_\tau$ is defined by induction on the complexity of τ:

1) $\mathcal{F}_{\boldsymbol{o}} \leftrightharpoons \mathcal{A}, \mathcal{F}_B \leftrightharpoons \mathcal{B}_2$
2) $\mathcal{F}_{(\tau_0 \times \tau_1)} \leftrightharpoons \mathcal{F}_{\tau_0} \times \mathcal{F}_{\tau_1}$
3) $\mathcal{F}_{(\tau_0 \to \tau_1)} \leftrightharpoons F(\mathcal{F}_{\tau_0}, \mathcal{F}_{\tau_1})$ □

If $\tau \in PTypes_f$ then $\mathcal{F}_\tau$ is an f^*-base.

Definition By a Σ-predicate of type $\tau \in Types_f$ on A we mean an arbitrary element of $I_\Sigma(\mathcal{F}_\tau)$. □

The propositions below easily follow from the definitions. Here $\Sigma(\mathbb{A})$ denotes the set of all Σ-definable subsets of $\mathbb{A}$.

Lemma For any $n > 0$ there is a natural bijective correspondence between Σ-predicates of type $\boldsymbol{o}^n \to B$ and n-ary Σ-predicates on $\mathbb{A}$. □

Proposition A mapping $F : \Sigma(\mathbb{A}) \to \Sigma(\mathbb{A})$ is a restriction of a Σ-operator if and only if F is continuous with respect to the strong topology and there is a Σ-function $f : A \to A$ such that $F(Q_{u,a}) = Q_{u,f(a)}$ for all $a \in A$. □

Proposition For a family $S \subseteq \Sigma(\mathbb{A})$ the following are equivalent:

1. S is represented by a Σ-predicate of type $((\boldsymbol{o} \to B) \to B)$
2. there is a Σ-formula $\Phi(P^+)$ of signature $\sigma \cup \langle P^1 \rangle$ such that

$$S = \{Q \mid Q \in \Sigma(\mathbb{A}), \langle \mathbb{A}, Q \rangle \models \Phi(P)\}$$

Proposition There is a natural bijective correspondence between Σ-predicates of type $((\boldsymbol{o} \to B) \to (\boldsymbol{o} \to B))$ and unary Σ-operators. □

We consider here the most natural case for studying algorithmic issues of Montague intensional logic, namely $\mathbb{A} = \mathbb{HF}(\mathbb{R})$. Indeed, the scale of time in linguislics is usually identified with the ordered set of real numbers $\mathbb{R}$. The correspondences in Table 2 were obtained in [3].

Table 2 Intensional Logic Types and $\mathbb{HF}(\mathbb{R})$

Category	Grammar equivalent	Type	Object in $\mathbb{HF}(\mathbb{R})$
e	no	e	sets $\{a\}$ for $a \in \mathbb{HF}(\mathbb{R})$
t	sentenses	t	no
IV	intransitive verbs	(e → t)	unary Σ-predicates
CN	common nouns	(e → t)	unary Σ-predicates
TV	extensional transitive verbs	(e → (e → t))	binary Σ-operators
CN/CN	extensional adjectives	((e → t) → (e → t))	Σ-operators
CN/CN	extensional adverbs	((e → t) → (e → t))	Σ-predicates
T	noun phrases and proper names	((s → (e → t)) → t)	Σ-definable families of binary Σ-predicates
t/t	sentence determiners	((s → t) → t)	Σ-definable families of Σ-predicates on $P_1(\mathbb{R})$
IV/t	connective verbs	((s → t) → (e → t))	Σ-operators

5 Rank and Vector Models of Intensional Logic

The main result of this paper about isomorphism of rank model and vector model provides a connection between two rather different methods of coding information. Both models are natural, from our point of view, the first because of the set-theoretical simplicity and the second because vectors or finite tuples are the typical approximations for infinite strings which are necessary to represent entities exactly.

Recall that a model of intensional logic is a tuple $\langle A, W, T, \leq, F\rangle$. Here we consider generalized computable models of intensional logic constructed with the help of computable functionals of finite types. In such models, the set A (entities of type e) corresponds to some f-base $\mathcal{A}$, the space of truth values (values of type t) corresponds to some f^*-base $\mathcal{B}$, the sets W and T used to form intensional types (states s) correspond to some f-base $\mathcal{W}$. The valuation function F corresponds to the entire hierarchy of functionals of finite types generated by the triple $\langle \mathcal{A}, \mathcal{B}, \mathcal{W}\rangle$ in accordance with the classical types considered in intensional logic. Thus, we say that the triple $\langle \mathcal{A}, \mathcal{B}, \mathcal{W}\rangle$ defines a (generalized computable) model of intensional logic.

As usual, $P_1(X)$ denotes the set of all one-element subsets of X, i.e., $P_1(X) = \{\{x\} \mid x \in X\}$. In [3] was introduced a model consisting of f-base

$$\mathcal{A}_0 = \langle X, =, P_1(X), \cup\rangle$$

for $X = HF(\mathbb{R}) \cup \mathbb{R}$ corresponding to the space of entities, f^*-base

$$\mathcal{B} = \langle \{0, 1\}, 0, \leq, \max\rangle,$$

corresponding to the space of truth values, and f-base

$$\mathcal{W} = \langle W, =, P_1(W), \cup\rangle$$

for $W = P_1(\mathbb{R})$ corresponding to the space of possible worlds. The entities in this model are singleton subsets of X and their structure is not taken into account: trivial equality is considered as a preorder on entities, and the entities themselves are simply "points" or "atoms". The space of truth values of this model can be intuitively interpreted as "0 means that the property does not exist, but may appear in the future" and "1 means that the property is and remains forever".

Also, in [3] was described a model that consists of f-base

$$\mathcal{A}_{vec} = \langle (\mathbb{R} \cup \{\bot\})^{<\omega}, \leq_{vec}, Cons_{vec}, \sqcup_{vec}\rangle,$$

where $\alpha_1 \leq_{vec} \alpha_2$ if and only if $lh(\alpha_1) \leq lh(\alpha_2)$ and $\alpha_1(i) \leq \alpha_2(i)$ for all $i \leq lh(\alpha_1)$, while we assume that $\bot \leq a$ for any $a \in \mathbb{R}$ and for $a, b \in \mathbb{R}$ are incomparable for $a \neq b$. Informally, entities are infinite tuples approximated via their finite initial subtuples, and contents of tuples correspond to properties (from categories IV and CN). In addition, we assume that some encoding is given, which says whether the i-th position of the ordered set is a binary or measurable property. For the space of truth values was used f^*-base

$$C = \langle \{0, 1, \bot, \top\}, \bot, \leq, \sqcup\rangle,$$

where $\bot < 0$, $\bot < 1$, $0 < \top$, $1 < \top$, and 0 and 1 are incomparable (elements of C stand for "no", "yes", "unknown" and "contradiction", correspondingly). Again,

$$\mathcal{W} = \langle W, =, P_1(W), \cup\rangle$$

for $W = P_1(\mathbb{R})$.

The triple $\langle \mathcal{A}_0, \mathcal{B}, \mathcal{W} \rangle$ will be called the *simplest model*, and the triple $\langle \mathcal{A}_{vec}, C, \mathcal{W} \rangle$ – *vector model* (for brevity, we will omit the space of possible worlds in what follows). In this section, some modification of the simplest model will be considered. It will take into account the structure of entities (that is, the elements that are contained in them as in sets). The entity $\{a\}$, $a \in HF(\mathbb{R})$, will be defined by a set of its properties, which are encoded by the set $\{a_1 \in HF(\mathbb{R}) \mid a_1 \in a\}$. A preorder relation on entities will be introduced, and the space of truth values will also be changed. The preorder relation and the space of truth values will be introduced in accordance with the vector model of intensional logic, so the resulting model (which we will call the *rank model* of intensional logic) will be isomorphic to it.

Consider the simplest model $\langle \mathcal{A}_0, \mathcal{B} \rangle$. Let us indicate a possible way of interpreting the properties of entities in this model. Consider $a \in HF(\mathbb{R})$: let $a = \{a_1, \dots, a_k\}$. The elements $a_1, \dots, a_k$ can be considered as properties of the object $\{a\}$ from the basic categories IV and CN. These properties can be decoded based on the ranks of the elements and some of their numerical characteristics.

Let us set $CN = \{cn_1, \dots, cn_n, \dots\}$, $IV = \{iv_1 \dots, iv_n, \dots\}$. All IV properties are binary (either hold or not), but CN properties can be either binary or take an arbitrary value from real numbers (for example, such properties as height or weight), so we will consider two different categories of CN_{bin} and CN_{cont}. Let us indicate a (possible) encoding of properties by natural numbers. If we associate with each category and element of this category the number ($IV \mapsto 1, iv_n \mapsto n; CN_{bin} \mapsto 2, (cn_{bin})_n \mapsto n, (cn_{bin})_n \mapsto n; CN_{cont} \mapsto 3, (cn_{cont})_n \mapsto n$), then using the Cantor function $c : \mathbb{N}^2 \to \mathbb{N}$, $c(x, y) = \frac{(x+y)^2+3x+y}{2}$, for the number $n \in \mathbb{N}$ we can restore a category and an element of this category. Let, for example, $IV = \{walk, talk\}$, $CN_{bin} = \{man, woman\}$ (we assume that they are numbered in the order they are listed). Since $c(1, 2) = 7$ and $c(2, 1) = 8$, we get that 7 corresponds to the category IV and the property $talk$, and 8 corresponds to the category CN and the property man. The numerical value of measurable properties can be encoded, for example, using the maximum real number contained in the support of the element a. More generally, we assume that some abstract Σ-function $Val : A \to \mathbb{R}$ is given, which determines the value for measurable properties.

As an example, consider the following model:

- $IV = \{talk\}$
- $CN_{bin} = \{human, male\}$
- $CN_{cont} = \{height, speed\}$

And the following match:

- $walk \mapsto c(1, 1)$
- $human \mapsto c(2, 1)$, $male \mapsto c(2, 2)$
- $height \mapsto c(3, 1)$, $speed \mapsto c(3, 2)$

Suppose that for some element $a = \{a_1, \dots, a_5\} \in HF(\mathbb{R})$ the following correspondence is given:

- $rnk(a_1) = c(1, 1)$

- $rnk(a_2) = c(2,1), rnk(a_3) = c(2,2)$
- $rnk(a_4) = c(3,1), Val(a_4) = 180, rnk(a_5) = c(3,2), Val(a_5) = 6$

The element $a = \{a_1, \ldots, a_5\}$ can be visualized using a graph as in Fig. 1.

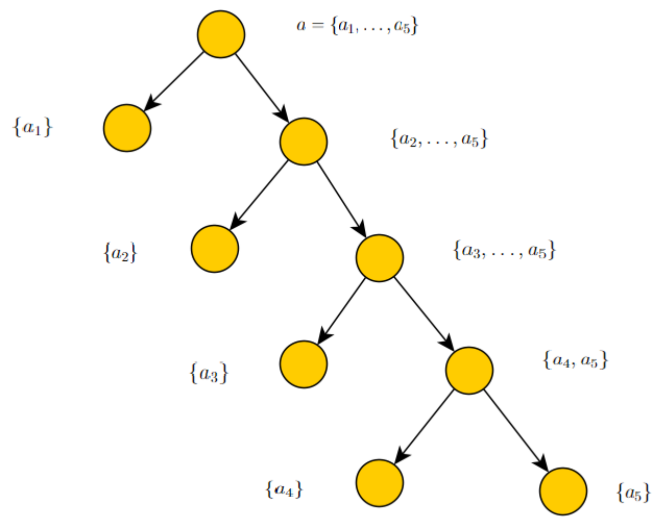

Fig. 1 Rank Tree

Then, looking at each vertex of the graph (for example, by breadth- or depth-first searches), we can conclude that

- $a_1 \sim walk$
- $a_2 \sim human$, $a_3 \sim male$
- $a_4 \sim height = 180$, $a_5 \sim speed = 6$

Using this correspondence, one can define Σ-predicates and Σ-operators as follows:

- $(a \in human) \Leftrightarrow (\exists x \in a)(rnk(x) = c(2,1))$
- $(a \in male) \Leftrightarrow (\exists x \in a)(rnk(x) = c(2,2))$
- $(a \in man) \Leftrightarrow (a \in male\ \ \&\ \ a \in human)$
- $(a \in tall(man)) \Leftrightarrow (a \in man\ \ \&\ \ (\exists x \in a)(rnk(x) = c(3,1)\ ;\ \&\ \ Val(x) \geq 185))$
- $(a \in slowly(walk) \Leftrightarrow (a \in walk\ \ \&\ \ (\exists x \in a)(rnk(x) = c(3,2)\ \ \&\ \ Val(x) \leq 3))$

5.1 Isomorphism between Rank and Vector Models

The reasoning above was given within the framework of the simplest model. The vector (or ontological) model of intensional logic uses a different approach to compare entities and different spaces of truth values.

Denote by Nat the set of ordinals in $HF(\mathbb{R})$. Let's assume that some encoding of categories IV and CN is given (for example, as specified in the previous subsection). There exists a partial Σ-function $\nu : Nat \to Nat^2$, which allows one to effectively determine a category and a property from this category by number, and its domain of definition is a Δ-set, as well as some Σ-function $Val : HF(\mathbb{R}) \to Nat$, which determines the numerical characteristics of properties from CN_{cont}. Accordingly, there are Δ-formulas $\varphi_{bin}(x)$ and $\varphi_{cont}(x)$ such that $\varphi_{bin}(x) \Leftrightarrow$ "x matches the binary property " and $\varphi_{cont}(x) \Leftrightarrow$ "x matches the continuous property ". We indicate (following the definition of the vector model) how the presence, absence, or uncertainty of a property can be interpreted in the rank model. If $x \in a$ for $a \in HF(\mathbb{R})$ then:

- if $0 \in x$, then we assume that the property (determined by the rank of the element x) is missing;
- if $0 \notin x$, then we assume that the property is present;
- if an element of rank n is absent in a, then we assume that we do not know about the presence of this property.

In view of what has been said, it is necessary to exclude some of the elements from $HF(\mathbb{R})$ in order to avoid ambiguous interpretation. Namely, to exclude all elements that contain (as sets) different elements of the same rank, as well as elements whose rank does not belong to the domain of the encoding function ν. Thus, we will consider a set S such that

$$a \in S \Leftrightarrow \forall x \in a[\forall y \in a(rnk(x) \neq rnk(y)) \,\&\, rnk(x) \in dom(\nu)].$$

S is a Δ-set. Denote by $rnk^*(a)$ the set of ranks of elements from a, i.e., $rnk^*(a) = \{n \in Nat : \exists x \in a(rnk(x) = n)\}$. On the set S, we introduce the Δ-preorder $\leq_1$ as follows:

$$a_1 \leq_1 a_2 \Leftrightarrow rnk^*(a_1) \subseteq rnk^*(a_2) \,\&\, \forall x \in a_1[(0 \in x \to \exists y \in a_2(rnk(x) = rnk(y))$$

$$\&\, 0 \in y))\&(0 \notin x \,\&\, \varphi_{cont}(x) \to \exists y \in a_2(rnk(x) = rnk(y) \,\&\, Val(x) = Val(y)))].$$

The equivalence relation defined by this preorder makes it possible to consider equal elements with the same properties, but in which these properties are defined by different elements. Therefore, in what follows we will consider the sets of equivalence classes $[S]$ and the (induced on it) order $\leq_1$.

Recall that the elements of the set $Cons$ for the vector model are finite tuples from $(\mathbb{R} \cup \{\perp\})^{<\omega}$. The $\sqcup$ function is clearly defined. Let's denote them by $Cons_{vec}$ and $\sqcup_{vec}$ respectively. As the set $Cons$ for the rank model, we also consider finite tuples (from $[S]$), and the function $\sqcup$ is defined similarly to a vector one. Let's denote them by $Cons_{rnk}$ and $\sqcup_{rnk}$, respectively. The orders on the vector and rank models will also be denoted by $\leq_{vec}$ and $\leq_{rnk}$. Consider f-bases

$$\mathcal{A}_0 = \langle [S], \leq_{rnk}, Cons_{rnk}, \sqcup_{rnk} \rangle,$$

$$\mathcal{A}_{vec} = \langle (\mathbb{R} \cup \{\bot\})^{<\omega}, \leq_{vec}, Cons_{vec}, \sqcup_{vec} \rangle$$

, and the mapping $\beta : \mathcal{A}_0 \to \mathcal{A}_{vec}$ defined as follows: the element $a \in [S]$ corresponds to an ordered set α of length $rnk(a) - 1$ such that for all $i \leq lh(\alpha)$:

- if $i \notin rnk^*(a)$ then $\alpha(i) = \bot$
- if $\exists x \in a(rnk(x) = i \,\&\, 0 \in x)$ then $\alpha(i) = 0$
- if $\exists x \in a(rnk(x) = i \,\&\, 0 \notin x \,\&\, \varphi_{cont}(x))$ then $\alpha(i) = Val(x)$
- else $\alpha(i) = 1$

Let us show that β is a bijection. Let's show injectivity. Let $a_1, a_2 \in [S]$ and $a_1 \neq a_2$, $\beta(a_1) = \alpha_1, \beta(a_2) = \alpha_2$. Either $rnk^*(a_1) \neq rnk^*(a_2)$ or $rnk^*(a_1) = rnk^*(a_2)$. The first immediately implies $\alpha_1 \neq \alpha_2$, since there is an element x of rank i such that $x \in a_1$ and $x \notin a_2$, so by the definition of the mapping β $\alpha_1(i) \neq \alpha_2(i)$ (or $\alpha_2(i)$ is not defined at all and $lh(\alpha_1) \neq lh(\alpha_2)$). Let $rnk^*(a_1) = rnk^*(a_2)$. Then one of the following is required:

- $\exists x \in a_1 \exists y \in a_2(rnk(x) = rnk(y) \,\&\, 0 \in x \,\&\, 0 \notin y)$
- $\exists x \in a_1 \exists y \in a_2(rnk(x) = rnk(y) \,\&\, \varphi_{cont}(x) \,\&\, Val(x) \neq Val(y))$

Each item immediately follows $\alpha_1 \neq \alpha_2$, so β is an injection. Let's show surjectivity. Let $\alpha \in (\mathbb{R} \cup \{\bot\})^{<\omega}$. Consider an element $a \in [S]$ such that for all $i \leq lh(\alpha)$:

- if $\alpha(i) = \bot$ then $i \notin rnk^*(a)$
- if i corresponds to a continuum property and $\alpha(i) \neq \bot$ then $\exists x \in a(rnk(x) = i \,\&\, Val(x) = \alpha(i))$
- if $\alpha(i) = 0$ then $\exists x \in a(rnk(x) = i \,\&\, 0 \in x)$
- else $\exists x \in a(rnk(x) = i \,\&\, 0 \notin x)$

These conditions uniquely define an element $a \in [S]$ such that $\beta(a) = \alpha$, so β is a surjection.

Let us now show that the mapping β is order-preserving. Let $a_1, a_2 \in S$, $a_1 \leq_{rnk} a_2$, $\beta(a_1) = \alpha_1, \beta(a_2) = \alpha_2$. Since $a_1 \leq_{rnk} a_2$, then $lh(\alpha_1) \leq lh(\alpha_2)$. Let $i \leq lh(\alpha_1)$. If $\alpha_1(i) = \bot$, then $\alpha_1(i) \leq \alpha_2(i)$. If $\alpha_1(i) = 0$, then $\exists x \in a_1(rnk(x) = i \,\&\, 0 \in x)$ is true by the definition of β, and by definition of order $\leq_1$, we get that $\exists y \in a_2(rnk(x) = rnk(y) \,\&\, 0 \in y)$, which gives $\alpha_2(i) = 0$. If now i corresponds to a continuum property, then $\exists x \in a_1(rnk(x) = i \,\&\, \varphi_{cont}(x) \,\&\, Val(x) = \alpha_1(i))$, which gives $\exists y \in a_2(rnk(x) = rnk(y) \,\&\, \varphi_{cont}(x) \,\&\, Val(y) = \alpha_1(i))$, and therefore $\alpha_1(i) = \alpha_2(i)$. So $\alpha_1(i) \leq \alpha_2(i)$ is true for all $i \leq lh(\alpha_1)$, and by definition of the order $\leq_{vec}$ we get that $\alpha_1 \leq_{vec} \alpha_2$. Thus, we have

Proposition There exists an isomorphism between f-bases $\mathcal{A}_0 = \langle [S], \leq_{rnk}, Cons_{rnk}, \sqcup_{rnk} \rangle$ and $\mathcal{A}_{vec} = \langle (\mathbb{R} \cup \{\bot\})^{<\omega}, \leq_{vec}, Cons_{vec}, \sqcup_{vec} \rangle$. □

If we take

$$C = \langle\{0, 1, \bot, \top\}, \bot, \leq, \sqcup\rangle,$$

as the truth space, then we get the following

Corollary Hierarchies of computable functionals of finite types for a rank model of the form $\langle\mathcal{A}_0, C\rangle$ and a vector model of the form $\langle\mathcal{A}_{vec}, C\rangle$ are equivalent. □

Moreover, from the definition of this isomorphism it is clear that it is a Σ-function.

The ranks of ordered sets of parent elements depend only on the length of the set. Indeed, the following (more general) fact is true

Lemma If $\mathbb{A}$ is an admissible set and $a_1, \ldots, a_n \in \mathbb{A}, n \geq 2$ then $rnk(\langle a_1, \ldots, a_n\rangle) = \sup\{\sup\limits_{i=\overline{1,n-1}}\{rnk(a_i) + 2i\}, rnk(a_n) + 2(n-1)\}$. □

Proof Induction on n. For $n = 2$ the assertion is true.

$n \longrightarrow n+1$: since $\langle a_1, \ldots, a_{n+1}\rangle = \langle a_1, \langle a_2, \ldots, a_{n+1}\rangle\rangle$, then

$$rnk(\langle a_1, \ldots, a_{n+1}\rangle) = \sup\{rnk(a_1) + 2, \sup_{i=\overline{2,n}}\{rnk(a_i) + 2i\}, rnk(a_{n+1}) + 2n\} =$$

$$= \sup\{\sup_{i=\overline{1,n}}\{rnk(a_i) + 2i\}, rnk(a_{n+1}) + 2n\}.$$

Hence, if $r_1, \ldots, r_n \in \mathbb{R} \cup \{\bot\}$ then $rnk(\langle r_1, \ldots, r_n\rangle) = 2(n-1)$. In the rank model, by definition, the rank of an element corresponding to an ordered set of length n does not exceed n. Thus, even for $n > 2$, elements of lower ranks are obtained. In the general case, the ranks of the elements of the rank model will be much smaller, since if $r_i = \bot$, then the element of rank i is absent in the corresponding object of the rank model. In addition, due to the introduced equivalence relation when defining the rank model, most of the content of the hereditarily finite superstructure becomes insignificant. In particular, since the rank (but not the content) uniquely determines some property, it is sufficient to confine ourselves to considering only one parent element.

6 Interpreting the Semantics of Possible Worlds

In this section, we will show how, for a possible world given by a real number (more precisely, the set $w = \{r\}$, where $r \in \mathbb{R}$), one can reasonably define a Δ-subset in $HF(\mathbb{R})$ and an f-base on this Δ-subset, which will be an interpretation of possible world w. Having such f-bases for all possible worlds, when considering questions of the truth of formulas (depending on the possible world), one can switch from using universal valuation, which, depending on the possible world, assigns one or another value to a variable, to checking the truth of formulas in the constructed f-bases, which are already significantly smaller than the original structure. In addition, this allows us to consider the entire hierarchy of computable functionals of finite types already on the structure corresponding to the possible world.

Various interpretations will be given for f-bases

$$\mathcal{A}_0 = \langle X, =, P_1(X), \cup \rangle,$$

where $X = HF(\mathbb{R})$, and

$$\mathcal{A}_{vec} = \langle (\mathbb{R} \cup \{\bot\})^{<\omega}, \leq, Cons_{vec}, \sqcup_{vec} \rangle$$

due to their fundamental differences.

Here we indicate how, given a real number $r \in \mathbb{R}$ in a hereditarily finite superstructure $\mathbb{HF}(\mathbb{R})$, one can define a countable Δ-definable set of real numbers. To do this, we need several auxiliary Σ-functions, defined (by Σ-recursion over ordinals) as follows:
$f_1 : Nat \to \mathbb{R}$, $f_1(0) = 0$, $f_1(n+1) = f(n) + 1$.
$f_2 : \mathbb{R} \times Nat \to \mathbb{R}$, $f_2(r, 0) = r$, $f_2(r, n+1) = \frac{f_2(r,n)}{10}$.
$pr : \mathbb{R} \times Nat \to Nat$, $pr(r, 0) = a_0 \Leftrightarrow a_0 \in Nat \,\&\, f_1(a_0) \leq r \leq f_1(a_0) + 1$, $pr(r, n+1) = a_{n+1} \Leftrightarrow a_{n+1} \in Nat \,\&\, \frac{f_1(a_{n+1})}{10^n} \leq x - \sum_{i=0}^{n} \frac{f_1(a_i)}{10^i} \leq \frac{f_1(a_{n+1})}{10^n} + 1$.
The function f_1 associates a natural number (ordinal) with a real number (primary element) corresponding to this natural number. The function f_2 divides the real number r by 10 to the power of n, and the function pr determines the natural number corresponding to the n-th digit in the decimal representation of the real number r. From the function pr one can define (by Σ-recursion) the Σ-function $f_3 : \mathbb{R} \times Nat \to Nat$, which enumerates the sequence of the first n numbers in the decimal representation of a real number , i.e., if $r = a_0, a_1 \dots a_{n-1} \dots$, then $f_3(r, n)$ is the number of the sequence of natural numbers $\langle a_0, \dots a_{n-1} \rangle$.

Let some natural number k be given, which indicates how many characters in the decimal representation of real numbers to consider. We will call this number the order of approximation of real numbers. Let $r \in \mathbb{R}$. We define the Δ-set $S_{\{r\}}$ as follows:

$$x \in S_{\{r\}} \Leftrightarrow \exists l (f_3(x, k) = l \,\&\, \forall i \leq k (pr(x, i) = pr(r, p_l^i)) \,\&\, \exists n (n \in Nat \,\&$$

$$\& \, f_1(n) = x \cdot 10^k),$$

$$x \notin S_{\{r\}} \Leftrightarrow \exists l (f_3(x, k) = l \,\&\, \exists i \leq k (pr(x, i) \neq pr(r, p_l^i)) \vee \exists n (n \in Nat \,\&$$

$$\& \, f_1(n) < x \cdot 10^k < f_1(n) + 1),$$

where p_l is the lth prime number. Thus, the set $S_{\{r\}}$ will include all real numbers of the form $a_0, a_1 \dots a_{k-1}$ whose i-th digit in decimal representation is equal to p_l^ith digit in decimal representation of r, where l is the sequence number $\langle a_0, \dots, a_{k-1} \rangle$. In addition, we define the Σ function $\nu_{\{r\}} : S_{\{r\}} \to Nat$ as follows: $\nu(x) = n \Leftrightarrow x \in S_{\{r\}} \,\&\, \forall i \leq k (pr(x, i) = pr(r, p_n^i))$. This function enumerates the set $S_{\{r\}}$.

6.1 Interpretation of Possible Worlds Semantics for Simplest Model

Consider the simplest model

$$\mathcal{A}_0 = \langle X, =, P_1(X), \cup \rangle,$$

where $X = HF(\mathbb{R})$. The possible world is given by an element from $P_1(\mathbb{R})$, i.e., set of the form $w = \{r\}$ for $r \in \mathbb{R}$. Based on the possible world w, we define the Δ-set S_w as in the previous section. Based on it, we construct the Δ-set H_w as follows:

$$x \in H_w \Leftrightarrow sp(x) \subseteq S_w,$$

where $sp(x)$ is the support of x. It is clear that this is also a Δ-set. Then on the set H_w it is possible to define an f-base

$$\mathcal{H}_w = \langle H_w, =, P_1(H_w), \cup \rangle,$$

which we will consider as an f-base, corresponding to the possible world w.

Intuitively, one can consider real numbers as some initial "filling" of $\mathcal{A}$, which contains information about all possible worlds at once. Sets from $HF(\mathbb{R})$ are constructed over this filling, i.e., objects of our structure $\mathcal{A}$. Choosing a real number r and considering the set $S_{\{r\}}$ specified by it, we select a part from the entire content of the structure $\mathcal{A}$ and consider the objects built on this part, which are the objects of the possible world $w = \{r\}$.

6.2 Interpretation of Semantics of Possible Worlds for Vector Model

Let us interpret possible worlds for a vector model of the form

$$\mathcal{A}_{vec} = \langle (\mathbb{R} \cup \{\perp\})^{<\omega}, \leq_{vec}, Cons_{vec}, \sqcup_{vec} \rangle.$$

Similarly, we consider the set $w = \{r\}$ as a possible world and construct a Δ-set S_w from it. Given the set S_w, we define the Δ-set H_w as follows (assuming that only ordered sets are considered):

$$x \in H_w \Leftrightarrow \forall i \leq lh(x)[\exists r(r \in S_w \,\&\, \nu_w(r) = i \,\&\, x(i) \in S_{\{r\}}) \vee$$

$$\vee (i \notin pr_r^*(\nu_w) \,\&\, \text{“}x(i) \text{ takes any valid value”})],$$

where the formula in quotation marks at the end is written as in Sect. 6.2 using the Σ-formulas φ_{bin} and φ_{cont}. It is also clear that the complement of the set H_w is a Σ-set, so it is indeed a Δ-set. Restrictions of the order $\leq_{vec}$, the set $Cons$, and the function $\sqcup$ to H_w allow us to correctly define the f-base

$$\mathcal{H}_w = \langle H_w, \leq_{vec} \cap (H_w)^2, Cons_{vec} \cap (H_w)^*, \sqcup_{vec} \rangle,$$

which will be treat as an interpretation of the possible world w.

The algorithm for constructing a support for a possible world consists in constructing a countable set of real numbers S_w from the possible world $w = \{r\}$. Further, we assume that each number $r_1 \in S_w$ together with its number $i = \nu_w(r_1)$ and a similar countable set of real numbers $S_{\{r_1\}}$ defines the set of admissible values for all i-th coordinates of ordered sets. Next, the set H_w contains all such ordered sets whose coordinate values belong to the sets of their admissible values. The set H_w is considered to be the set of objects in the possible world w.

To analyze time processes, one has to consider both points (moments) and intervals of the form (r_1, r_2), where $r_1 \leq r_2$ are real numbers. Using the above Σ-functions for encoding real numbers and a note about the order of approximation of real numbers, as well as some way of encoding ordered pairs of real numbers (for example, the pair $r_1 = x, x_1x_2\dots, r_2 = y, y_1y_2\dots$ corresponds to the number $r = 0, xyx_1y_1x_2y_2\dots$) we get that there are Σ-predicates $cint(r)$ $int(r, r_1, r_2)$ true if and only if the real number r encodes some interval and when the real number r encodes the interval (r_1, r_2) respectively. Further, following the already defined constructions, we set $S^{int}_{\{r\}} = S_{\{r_1\}} \cup S_{\{r_2\}}$ as a possible world for the interval (r_1, r_2) given by the number r. If it is necessary to use time intervals or a point in time, the possible world $w = \{r\}$ can be treated in one way or another.

6.3 Analysis of Past Simple, Future Simple and Present Continuous in English

Let us give examples of using the interpretation of possible worlds. Consider sentences

(1) Michele barked,

(2) Michele will bark,

(3) Michele is barking,

as well as

(4) Michele barks.

It is clear that on possible worlds one can define by a Δ-formula an order relation consistent with the order on real numbers (i.e., $w_1(=\{r_1\}) \leq w_2(=\{r_2\}) \Leftrightarrow r_1 \leq r_2$). In [3] is given an analysis of English sentences using object intensions. Here, we will consider m as a variable with a certain set of its possible values (extensions). Moreover, for each Σ-subset S in $HF(\mathbb{R})$ there is a Σ-subset $S_w \subseteq S$ in H_w (the support of f-base $\mathcal{H}_w$ corresponding to the possible world w). This Σ-subset is obtained in the same way as Σ-subsets were obtained from the Σ-ideals of the functional product $F(\mathcal{A}, \mathcal{B})$, with the replacement by f-base $\mathcal{A}$ to f-base $\mathcal{H}_w$. Then the Σ-subset S_w can be considered an interpretation of the most general Σ-subset S in the possible world w.

Therefore, proposition (1) is true in the model $\mathcal{A}$ and the possible world $w = \{r\}$ if

$$\mathcal{A} \models \exists w_1(=\{r_1\})\exists r_2 r_3(int(r_1, r_2, r_3)\,\&\, r_3 < r \,\&\, m \in S^{int}_{w_1} \,\&\, bark'_{w_1}(m))$$

(when valuing γ), where $bark'(x)$ is the Σ-predicate corresponding to the verb *bark*. In other words, sentence (1) is true at time r if there exists a time interval (r_2, r_3) such that $r_3 \leq r$ and sentence (4) is true in this time interval. Proposition (2) is treated similarly, with a change of order. Proposition (3) is true in the model $\mathcal{A}$ and the possible world $w = \{r\}$ if

$$\mathcal{A} \models \exists w_1(=\{r_1\})\exists r_2 r_3(int(r_1, r_2, r_3)\,\&\, r_2 < r < r_3 \,\&\, m \in S^{int}_{w_1} \,\&\, bark'_{w_1}(m)).$$

In the case of proposition (3), the possible world w for which the truth of the proposition is checked can also be an interval if necessary.

Algorithmic issues of interval semantics for Perfect tenses in English and the category of aspect in Russian are discussed in [17, 16].

It is interesting that, in contrast to extensional objects, intensional issues (more exactly, temporal aspects) are arranged very different in English and in Russian. We discuss these differences in one of the forthcoming papers.

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