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DYNAMICS OF BREAKING ZONE DEVELOPMENT DURING EXPLOSION OF A CONCENTRATED CHARGE IN A BRITTLE MEDIUM

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The deformation and failure of brittle monolithic rocks under the action of confined explosion of a concentrated charge are examined. A calculated scheme is suggested for determining the dynamics of breaking zone development. Features of the increase in radial crack area are considered.

The theory of the explosive action of a concentrated charge in rock in a quasi-static statement has been developed in [1, 2]. A model including precise dynamic description of the development of breaking during an explosion was provided in [3]. A zonal approach was used in these works for describing medium deformation. Close to the explosion cavity there is normally a zone of shear crushing, and beyond it there is a zone of radial cracks (columnar elasticity) and then an outer zone of elasticity.

The main difference in the present work from those indicated is the use of failure condition for elastic medium at the radial crack front taking account of the energy expended on forming new surfaces. This criterion was proposed in [4] and clarified in [5]. There is another important difference that involves using different functions for principal stresses as elastic medium failure conditions at the boundary with the zone of crushing and plasticity within the latter. Ranking of parameters with respect to their effect on the maximum dimensions of the breaking zone is carried out.

STATEMENT OF THE PROBLEM

The deformation and failure of a brittle isotropic medium under the action of explosion of a concentrated explosive charge are examined. Medium deformation after charge detonation may be separated into several stages. The first is expansion of the explosion cavity and propagation of the crushing zone with a velocity exceeding the maximum velocity of crack development v_{max} that is characteristic for the medium being failed. One elastic zone $(r \ge a)$, or two zones, i.e., plastic $(a \le r \le b)$ and elastic $(r \ge b)$, are possible, where $a(t)$ is explosion cavity radius, $b(t)$ is crushing zone boundary, and r is the current radius. In the second stage with $b < v_{max}$ a radial crack zone appears $(b \le r < l,$ where $l(t)$ is crack zone front radius). Calculation of deformation within zones of elasticity and cracks is performed in a quasi-static approximation. Stresses and strains within them are determined from static relationships for loads at the current instant of time. Crack density within the columnar elasticity zone is a supplementary parameter of the process and it determines the number N of columnar jointings separated by them transferring pressure from grinding zone of the elastic zone. Analysis of experimental data showed that with development of a radial crack zone their density falls and the number of columnar jointings decreases. In order to evaluate the possible crack density at different distances from the explosion center, calculations of the second stage were carried out for a number of values of N, for example 64, 32, 16, 8, and 4. A similar statement of the problem was used in [6] in describing breaking of brittle medium under the action of a cylindric charge.

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CALCULATION MODEL

We write the defining relationships, equations of motion, and general solutions for each zone.

Gas pressure in the explosion cavity is calculated from a modified Jones – Miller adiabat [1] for a spherical charge

$$p(a)=\begin{cases} p_0\left(\dfrac{a}{a_0}\right)^{-3\gamma_1}, & a\le a^*, \\ p_0\left(\dfrac{a^*}{a_0}\right)^{-3\gamma_1}\left(\dfrac{a}{a^*}\right)^{-3\gamma_2}, & a\ge a^*, \end{cases} \tag{1}$$

where $\gamma_1=3$, $\gamma_2=1.27$, $a/a^*=1.53$. For trotyl p_0=10^{10} Pa.

In the elastic zone in the case of spherical symmetry we obtain a general solution for radial displacement $u(r, t)$ with respect to the unloaded state of the medium

$$u(r,t)=\frac{f'(t-r)}{r}+\frac{f(t-r)}{r^2}-\frac{(1-\nu)}{(1+\nu)}Pr, \tag{2}$$

derived on dimensionless coordinates with a scale for length a_0, time a_0/c_0, $(c_0^2=E_1/\rho_0)$ and stress

$$E_1=E\frac{1-\nu}{(1+\nu)(1-2\nu)}.$$

Here, E is Young's modulus, ν is Poisson's ratio, ρ_0 is elastic medium density, and P is external pressure. The stress tensor components of this solution in dimensionless form are expressed in terms of arbitrary function f as follows:

$$\begin{aligned} \sigma_r &=-\frac{1}{r}f''(t-r)-4\mu^2\left[\frac{1}{r^2}f'(t-r)+\frac{1}{r^3}f(t-r)\right]-P, \\ \sigma_\theta &=-\frac{1-2\mu^2}{r}f''(t-r)+2\mu^2\left[\frac{1}{r^2}f'(t-r)+\frac{1}{r^3}f(t-r)\right]-P, \end{aligned} \tag{3}$$

where $\mu^2=0.5(1-2\nu)/(1-\nu)$.

With a quasi-static consideration for the elastic zone loaded by pressure $(-q)$ at the cavity radius b and by pressure P at infinity we have:

$$\begin{aligned} \sigma_r &=-P+(q+P)b^3/r^3, \\ \sigma_\theta &=-P-0.5(q+P)b^3/r^3, \\ u_r &=-(1+\nu)(q+P)b^3/(2Er^2)-(1-2\nu)Pr/E. \end{aligned} \tag{4}$$

The Coulomb criterion is used as a breaking criterion for material of the elastic zone with $\sigma_r<0$ determining in plane $(\sigma_r, \sigma_\theta)$ the region of elastic medium behavior. With $\sigma_r<\sigma_\theta$ this region is bounded by a straight line

$$(1+\alpha_2)\sigma_\theta-\sigma_r-Y_2=0, \tag{5}$$

and parameters α_2 and Y_2 may be determined from uniaxial tests in tension σ_t and compression σ_c

$$\alpha_2 = \frac{\sigma_c}{\sigma_t} - 1, \quad Y_2 = \sigma_c .$$

For hard rocks data for the values vary: $\alpha_2 = 7 - 12$, $Y_2 = (0.6 - 2.7) \cdot 10^8$ Pa.

In the region of stresses, where $\sigma_r > 0$ and $\sigma_\theta > 0$, the boundaries of the breaking zone are $\sigma_r = \sigma_t$ and $\sigma_\theta = \sigma_t$. A similar criterion was used in [3] and it has been confirmed for rocks [7].

In the crushing zone close to the charge it is assumed that equations of flowing medium motion are fulfilled. For the one-dimensional case of central symmetry we obtain

$$\rho\left(\frac{\partial \mathrm{v}}{\partial t} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial r}\right) = \frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_\theta)}{r}, \tag{6}$$

here, ρ is medium density, v is radial velocity. As a defining relationship we take the Coulomb rule [1] $\tau = C - \sigma \mathrm{tg}\varphi$, where τ and σ are tangential and normal stresses in the shear area, C and φ are medium parameters. In principal stresses of the centrally-symmetrical problem with explosion cavity expansion in this zone we have

$$(1+\alpha)\,\sigma_\theta - \sigma_r - Y = 0, \tag{7}$$

$$Y = 2C\cos\varphi/(1-\sin\varphi), \quad \alpha = 2\sin\varphi/(1-\sin\varphi).$$

When the cavity is compressed the yielding condition acquires the form $(1+\alpha)\sigma_r - \sigma_\theta - Y = 0$ that may be formally reduced to (7) with new α_1 and Y_1

$$(1+\alpha_1)\sigma_\theta - \sigma_r - Y_1 = 0,$$

$$Y_1 = -Y/(1+\alpha), \quad \alpha_1 = -\alpha/(1+\alpha).$$

Medium density ρ in (6) may be determined by material compression in the elasticity zone and subsequent compaction or loosening caused by the effect of dilation in the crushing zone. In order to describe this effect, we shall use a simple assumption [2] about the connection of volume strains with shear strains $\dot{\varepsilon} = \Lambda\dot{\Gamma}$. Whence a dilation condition follows

$$\frac{\partial \mathrm{v}}{\partial r} + 2\frac{\mathrm{v}}{r} = \Lambda\left(\frac{\mathrm{v}}{r} - \frac{\partial \mathrm{v}}{\partial r}\right),$$

here, Λ is the dilation coefficient.

As was demonstrated in [8], medium density $\rho(r, t)$ may be calculated from the continuity equation and dilation conditions in terms of movement parameters and current radius. Calculations showed that consideration of the change in ρ in (6) has little effect on the solution. Therefore in the present work medium density ρ in (6) is assumed to be constant and equal to the initial density ρ_0.

From dilation condition in the grinding zone it follows that

$$\mathrm{v}(r, t) = \dot{a}a^n / r^n, \tag{8}$$

where $n = (2 - \Lambda)/(1 + \Lambda)$.

Substituting σ_θ from (7) in (6) and integrating with respect to r taking account of (8), we obtain a general solution in the grinding zone

$$\sigma_r = \frac{Y}{\alpha} + \rho\left[S_1 \frac{(\dot{a}a^n)'_t}{r^{n-1}} - n S_2 \frac{(\dot{a}a^n)^2}{r^{2n}}\right] + G(t) r^{-2\alpha/(1+\alpha)}, \tag{9}$$

where $S_1 = \dfrac{1+\alpha}{(3-n)\alpha + (1-n)}$, $S_2 = \dfrac{1+\alpha}{2\alpha(1-n) - 2n}$, and $G(t)$ is an arbitrary function of time.

For the increment in displacement Δu_r in the crushing zone with $r \le b(t)$ from condition (8) with $t > t_\gamma$ it follows that

$$a^{n+1} - a_0^{n+1} = r^{n+1} - (r - \Delta u_r)^{n+1}.$$

Whence total displacement u_r in region $r \le b(t)$ has the form

$$u_r \approx u_r(b, t_r) + \frac{a^{n+1} - a^{n+1}(t_r)}{(n+1) r^n}, \tag{10}$$

here, t_r is the instant of time at which the crushing wave arrives at point r, and $u_r(b, t_r)$ is the displacement in the elastic zone at this point.

In the radial crack zone it is assumed that $\sigma_\theta = 0$. Within the framework of a quasi-static solution

$$\sigma_r = -p_b \frac{b^2}{r^2}, \qquad \frac{du}{dr} = -\frac{p_b b^2}{E r^r}, \qquad u = u_0(t) + \frac{p_b b^2}{Er}, \tag{11}$$

where p_b is radial pressure with $r = b$, u is radial displacement, $u_0(t)$ is an arbitrary function of time.

DEVELOPMENT OF THE CRUSHING WAVE

Breaking elastic medium after an explosion may spread with a compression wave or it may be delayed and arise after some time in the cavity or within the medium [3]. These versions are determined by the ratio of strength and elastic parameters of the medium, and the loading intensity for the explosion cavity. For hard rocks there is typically at first elastic expansion of the cavity and then development of breaking from the explosion cavity into the depth of the medium behind the compression wave.

Cavity radius $a(t)$, elastic potential $f(t)$, and its derivative $F(t) = f'(t)$ are determined in the model adopted according to (1) – (3) in the stage of elastic expansion of the cavity from the following set of equations

$$f' = F,$$

$$F' = p(a) - 4\mu^2[F - f], \tag{12}$$

$$a = 1 + f + F.$$

In the initial instant of time $(t = 1)$, $f = F = 0$.

Elastic development of the cavity continues until instant t_1 at which in the explosion cavity and then at the crushing wave front $r = b(t)$ breaking condition (5) will be satisfied, and hence (in view of (3)) we obtain: from instant of time t_1 in the crushing wave the following relationship is fulfilled

$$\frac{A}{b} f'' + \frac{B}{b^2} f' + \frac{B}{b^3} f - D = 0, \tag{13}$$

and correspondingly $A = \alpha_2 - 2\mu^2 - 2\alpha_2\mu^2$, $B = -2\mu^2(3 + \alpha_2)$, $D = -Y_2 - P\alpha_2$.

At boundary $r = b(t)$ separating the crushing and elasticity zones, stress σ_r and velocity v are taken as continuous. This approximation is permissible if the change in density in the crushing wave is small. The problem of crushing wave development within markedly compressed porous media has been examined in [9]. From the continuity conditions in the crushing wave and the explosion cavity $\sigma_r(b-0) = \sigma_r(b+0)$, $\frac{\partial u}{\partial t}(b-0) = \frac{\partial u}{\partial t}(b+0)$, $\sigma_r(a) = -p(a)$ and from solutions (2), (3), (8), (9), and (13) a set of equations follows for $a(t)$, $b(t)$, and $f(t-b)$

$$R_1 a\ddot{a} + n(R_1 - R_2)\dot{a}^2 + R_3 - p(a) = 0,$$

$$f''(t-b) = \frac{\dot{a}\, a^n}{b^{n-1}} - \frac{f'(t-b)}{b}, \tag{14}$$

$$\frac{A}{b} f''(t-b) + \frac{B}{b^2} f'(t-b) + \frac{B}{b^3} f(t-b) = D\ .$$

Here,

$$R_1 = -S_1[1-(b/a)^{1/S_1}], \quad R_2 = -S_2[1-(b/a)^{1/S_2}], \quad R_3 = -\frac{Y}{\alpha} + \left[\frac{Y}{\alpha} + p_b\right](b/a)^{2\alpha/(1+\alpha)},$$

$$p_b = \frac{\dot{a}a^n}{b^n} - \frac{f'(t-b)}{b^2} + 4\mu^2\left[\frac{f'(t-b)}{b^2} + \frac{f(t-b)}{b^3}\right] + P.$$

For functions $a(t)$, $V(t) = \dot{a}(t)$, $b(t)$, $\varphi(t) = f(t-b(t))$ and $\Phi(t) = f'(t-b(t))$ from (14) we obtain

$$\frac{da}{dt} = V, \tag{15}$$

$$\frac{dV}{dt} = \frac{p(a) - R_3 - n(R_1 - R_2)V^2}{aR_1}, \tag{16}$$

$$\frac{db}{dt} = \frac{A(Va^n)'_t + Q(Va^n/b) + A\Phi b^{n-3}}{Q(Va^n/b) + \Phi b^{n-3}[A(n-1) - B(n-2)] - B(n-3)b^{n-4}\varphi + nDb^{n-1}}, \tag{17}$$

$$\frac{d\varphi}{dt} = \Phi(1-\dot{b}), \tag{18}$$

$$\frac{d\Phi}{dt} = \left(\frac{Va^n}{b^{n-1}} - \frac{\Phi}{b}\right)(1-\dot{b})\ . \tag{19}$$

Here,

$$p_b = \frac{Va^n}{b^n} - \frac{\Phi}{b^2} + 4\mu^2\left[\frac{\Phi}{b^2} + \frac{\varphi}{b^3}\right] + P,$$

$$(Va^n)'_t = a^{n-1}[nR_2V^2 - R_3 + p(a)]/R_1,$$

$$Q = B - A.$$

At instant of time $t = t_1$ — $a = b = a(t_1)$, $V = V(t_1)$, $\varphi = f(t_1 - a(t_1))$, $\Phi = F(t_1 - a(t_1))$. Values of $\dot{V}$ and $\dot{b}$ are determined from set (15) – (19) if its solution is presented in the form of a series with respect to *t*. Thus, it is possible to reveal the uncertainty in Eqs (16) and (17) with $t = t_1$, when $b/a = 1$ and $R_1 = R_2 = 0$.

DEVELOPMENT OF THE RADIAL CRACK ZONE

During breaking of brittle medium by explosion a crushing wave may be overtaken the radial crack front. This occurs in calculations when tensile stresses σ_θ appear ahead of the crushing front and a condition is fulfilled with which the potential velocity of radial crack front development becomes greater than $\dot{b}$.

If the stressed state makes it possible for a sufficiently large number of radial cracks to break away from the breaking front (greater than some value of N_1), then by developing at a greater rate than the crushing front velocity they will divide the material ahead of the crushing front into radial rods, relieve from tension in the tangential direction, and thereby they will strengthen it for crushing. Here, the crushing front will stop at instant $t = t_2$ and breaking will develop due to development of a radial crack zone. If the number of cracks that have the crushing front velocity consistent with the load is insufficient for the relief of tangential tension ahead of the crushing front $(N < N_1)$, then it will develop according to (15) – (19) until *N* will comparable with N_1, and then there will be also stopping of the crushing front and development of a radial crack zone. Introduction of a threshold value of N_1 is a simplifying hypothesis for the solution scheme suggested. In the calculations it was assumed that $N_1 = 64$.

In the second stage of breaking for the three zones equations of type (15) – (19) follow from boundary conditions in the explosion cavity and at the boundaries of the three zones.

In the explosion cavity $r = a(t)$, $\sigma_r = -p(a)$; at the crushing zone boundary $r = b(t)$,

$$u(b-0) = u(b+0), \quad \sigma_r(b-0) = \sigma_r(b+0) = -\sigma_1 \qquad (\sigma_1 \le \sigma_c); \tag{20}$$

at the radial crack front $r = l(t)$,

$$u(l-0) = u(l+0), \quad \sigma_r(l-0) = \sigma_r(l+0) = q. \tag{21}$$

Equations (21) are approximate. In a precise statement [3] at the crack front there are discontinuities of velocity and stresses determined by a jump in density. Since for rocks they are small, then conditions (21) are valid. Calculations were carried out with the aim of verification for the dynamics of radial crack zone development using (21) in the problem solved in [10] with a precise dynamic condition at the crack front. The difference in the solutions was 5%.

With fulfilment of equality in the equation for stresses (20) there is crushing in the rod zone of radial cracks and $b(t)$ increases with time. In the opposite case $\sigma_r(b+0) > -\sigma_c$, the grinding zone boundary is not mobile, and $\dot{b} = 0$.

Using solutions (4) and (11) and excluding $u_0(t)$ from (20) and (21) in a quasi-static approximation for the radial crack and elastic zones we find the connection between the main parameters

$$\frac{u_b E}{bE_1} + \frac{3(1-\nu)Pl}{2b} - \sigma_1\left(1 - \frac{1-\nu}{2}\frac{b}{l}\right) = 0. \tag{22}$$

Having substituted solution (9) in boundary conditions in the explosion cavity and on the boundary of the grinding zone (20), after excluding *F*(*t*) we obtain an equation

$$R_1 a\ddot{a} + n(R_1 - R_2)\dot{a}^2 + R_3' - p(a) = 0. \tag{23}$$

Here,

$$R_3' = -\frac{Y}{\alpha} + \left[\frac{Y}{\alpha} + \sigma_1\right](b/a)^{\,2\alpha/(1+\alpha)}.$$

With

$$\sigma_1 = \frac{u_b E/(bE_1) + 3(1-\nu)Pl/(2b)}{1-(1-\nu)b/(2l)} < \sigma_c, \quad \dot{b} = 0.$$

If $\sigma_1 = \sigma_c$, then from (22) by differentiating with respect to *t* we find

$$\dot{b} = \frac{(1-\nu)\,\dot{l}(3P - \sigma_1 b^2/l^2) + 2(n+1)\,\dot{a}a^{n+1}/(3b^n)}{2\sigma_1(1-(1-\nu)\,b/l) + 2n(a^{n+1} - a^{n+1}(t_2))/(3b^{n+1})}\,. \tag{24}$$

For $l(t)$ in dimensionless form we have

$$\dot{l} = \begin{cases} 0, & \gamma \le \gamma_0, \\ \dfrac{v_{\max}}{c_0}\dfrac{1-\exp(-\beta_1(\sqrt{\gamma/\gamma_0}-1))}{1-\exp(-\beta_1(\sqrt{\gamma_1/\gamma_0}-1))}, & \gamma_0 < \gamma < \gamma_1\,, \\ \dfrac{v_{\max}}{c_0}, & \gamma \ge \gamma_1, \end{cases} \tag{25}$$

where γ is the current value of the specific surface energy of crack formation, γ_0 and γ_1 are the specific surface energies of crack formation with crack initiation from the start and the beginning of branching. Relationship (25) was proposed in [11] as an interpolation relationship for experimentally determined estimating relationships for a number of brittle media connecting crack resistance and crack development rate.

The current value of γ is determined from the energy condition at the radial crack front [5]. In the case of central symmetry and the quasi-static condition $\sigma_r^y = \sigma_r^c$ from [5] it follows that

$$2\gamma n = 0.5\sigma_r^c(\varepsilon_r^c - \varepsilon_r^y) + \varepsilon_\theta^y \sigma_\theta^y\,,$$

σ_r^y, σ_θ^y, ε_r^y, ε_θ^y are values of stresses and strains at the boundary of the elastic region $(r = l + 0)$, σ_r^c, ε_r^c are stresses and strains at the front of the rod zone of radial cracks $(r = l - 0)$, and n is the specific perimeter of crack contours at the surface of their front. By substituting here values of stresses and strains from (4) and (11) we obtain

$$2\gamma = (1-\nu)(q+3P)^2/(4nE)\,, \quad q = \sigma_r^c = -\sigma_1 b^2/l^2\,. \tag{26}$$

CALCULATIONS OF CRUSHING WAVE DYNAMICS

Calculations for breaking by set of Eqs. (15)–(19) obtained for the case of crushing wave propagation through elastic medium were carried out up to the instant of total stopping of the cavity expansion and the crushing wave. Values of these dimensions at the instant when the radial crack zone front emerges from the crushing wave front were used as initial data for calculations of the second stage of breaking for brittle media in the crack zone.

Set of Eqs. (15)–(19) was calculated numerically by the Runge–Kutta method. The following collection of parameters was used as a basis: $E = 5 \cdot 10^{10}$ Pa, $\rho = 2500$ kg/m^3, $\nu = 0.3$, $Y = 10^6$ Pa, $P = 10^5$ Pa, $Y_2 = 10^8$ Pa, $p_0 = 10^{10}$ Pa, $\alpha = 4$, $\alpha_2 = 9$, $\Lambda = 0$, $\gamma_1 = 120$ J/m^2, $\gamma_0 = 40$ J/m^2, $v_{max} = 830$ m/s, and $\beta = 1$.

Results of calculating the dependences $a(t)$, $b(t)$, $\varphi(t)$, and $\Phi(t)$ for the basic collection of parameters are presented in Fig. 1 as curves *1–4*, respectively. In this case the cavity radius increases monotonically up to a maximum value and then it remains constant. Another regime of explosion cavity dynamics is possible, which is distinguished by compression of the cavity and forms with a small friction angle of flowing medium in the cracking zone.

Characteristic dependences for explosion cavity radius on time t are provided in Fig. 2. Values of parameters $Y = 10^5$ Pa and $\alpha = 0.1; 0.2; 0.3; 1.0; 4.0$ (curves *1–5*) were used in the calculations. The rest of the parameters were chosen from the basic collection. Full curves correspond to calculation without radial crack zone, and dash-dotted curves correspond to consideration of their development. Depending on the magnitude of the internal friction angle for the medium in the grinding zone aperiodic and oscillatory regimes are possible for explosion cavity development. Return motion without the outflow of gases from the explosion cavity is only possible with quite small values of α ($\alpha < 1$) and friction angle ($\varphi < 20°$).

With the aim of clarifying the effect of the parameters of the problem on the maximum dimensions of the cavity a_m, the crushing zone b_m, and displacements at the boundary of this zone u_{bm}, a statistic statement of the problem was examined similar to that in [2] and [12] for the equilibrium size of the cavity with a grinding zone surrounding it in an elastic space under the action of detonation product gases in the explosion cavity. In order to determine the equilibrium values of a_s, b_s, and u_{bs}, we obtain a set of equations:

$$p(a_s) = -Y/\alpha + (Y/\alpha - q)(b_s/a_s)^{2\alpha/(1+\alpha)},$$

$$\frac{b_s}{a_s} = \left[\frac{(1-1/a^{n+1})}{3}\left(\frac{b_s}{u_{bs}}\right)\right]^{1/(n+1)},$$

$$q = -\frac{2Y_2 + 3(1+\alpha_2)P}{3+\alpha_2}, \tag{27}$$

$$\frac{u_{bs}}{b_s} = \frac{(1+\nu)(Y_2 + \alpha_2 P) - (3+\alpha_2)(1-2\nu)P}{(3+\alpha_2)(E/E_1)}.$$

With small values of P and Y/α ($P << 2Y_2/(3(1+\alpha_2))$, $(Y/\alpha << 2Y_2/(3+\alpha_2))$ this set is reduced to the equation

$$p(a_s) \approx 2\left(\frac{3(1+\nu)}{E/E_1}\right)^{\frac{-2\alpha}{(1+n)(1+\alpha)}}\left(\frac{3+\alpha_2}{Y_2}\right)^{\frac{2\alpha}{(1+n)(1+\alpha)}-1} = \Pi.$$

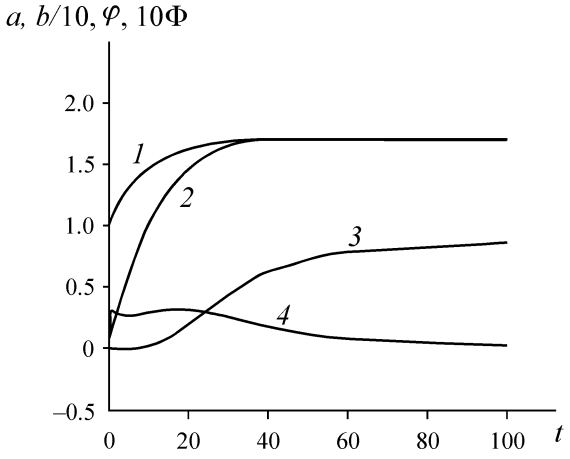

Fig. 1

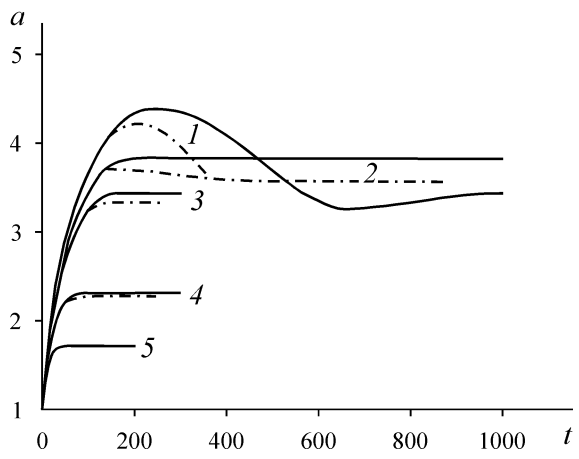

Fig. 2

Whence using (1) in dimensionless form

$$a_s \approx \begin{cases} \left(\dfrac{p_0}{\Pi}\right)^{1/3\gamma_1}, & a \le a^*, \\ \left(\dfrac{p_0}{\Pi (a^*)^{3(\gamma_1-\gamma_2)}}\right)^{1/3\gamma_2}, & a \ge a^*, \end{cases}$$

$$b_s \approx a_s \left[\frac{(3+\alpha_2)\,E/E_1}{3(1+\nu)Y_2}\right]^{1/(1+n)}.$$

Calculations with respect to the dynamic model for a_m, b_m and u_{bm} showed that within the range of changes in problem parameters indicated above with an accuracy up to 10% approximation relationships are fulfilled:

$$a_m = 1.99a_s - 0.98, \quad b_m = \widetilde{b}_s = b_s^{1.11-0.2\Lambda}, \quad u_{bm}/b_m = 0.88u_{bs}/b_s - 0.1. \tag{28}$$

This is illustrated by the plots in Figs. 3 – 5 in which results are provided for calculating (a_m, a_s), $(b_m, \widetilde{b}_s)$, $(u_{bm}/b_m, u_{bs}/b_s)$ for a number of collections of problem parameters and also approximating relationships. Taking account of these relationships, formulas (27) and (28) may be used for analytical evaluation of the maximum explosion cavity and grinding zone dimensions. We note that the maximum cavity and breaking zone dimensions obtained with solution of the dynamic problem exceed the static values by a factor of 0.9 – 1.3.

It was demonstrated in [6] that in order to evaluate the radial crack dimensions with explosion in brittle medium, there is marked expulsion of medium in the grinding zone into the depth of the medium determined by the maximum cavity radius and the size of the grinding zone in the stage of its expansion. In fact, during development of the latter crushed material in the expansion phase is dynamically expelled into the depth of the medium and after stopping it does not return to the centre of the explosion having packed loose material in the compression phase. This packing affects the breaking dimensions in the radial crack zone, since it governs the elastic medium close to static loading by an expanding piston ahead of the front of the crushing wave stopped.

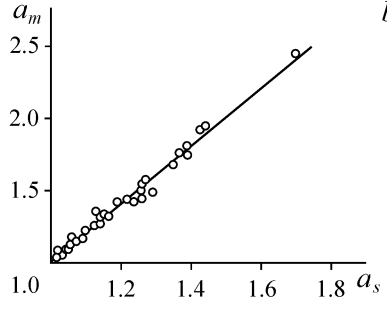

Fig. 3

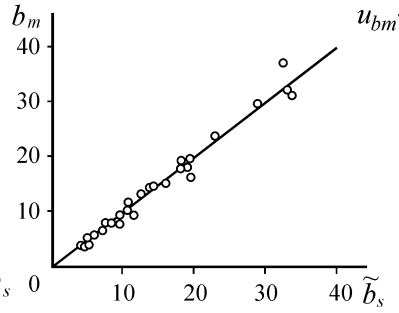

Fig. 4

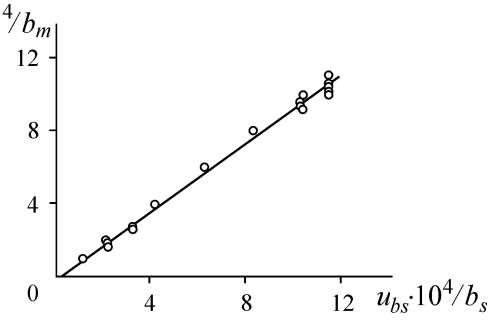

Fig. 5

STATIC EVALUATION OF BREAKING IN THE RADIAL CRACK ZONE FROM DATA FOR MEDIUM EXPANSION IN THE CRUSHING WAVE

If values of the crushing wave front radius b_k are known after its stopping and movements within it u_{bk}, it is possible to evaluate the limiting number N of columnar jointings cut out by radial cracks at distance l from the explosion centre. If the specific perimeter of cracks at a front of radius l equals n, then the cross-section area of a conical jointing at the front is of the order of $4/n^2$ and their number $N \approx \pi l^2 n^2$.

Using (26), we obtain

$$N = \pi l^2 \frac{(1-\nu)^2}{64\gamma_0^2 E^2}[q+3P]^4, \tag{29}$$

where

$$q = -\frac{b_k^2}{l^2}\left[\frac{u_{bk}E/b_k + 3(1-\nu)Pl/(2b_k)}{1-(1-\nu)b_k/(2l)}\right].$$

With $P << \dfrac{u_{bk}E}{9b_k l\,(1-\nu)}$ and $l >> b_k(1-\nu)/2$ it follows approximately that

$$\frac{l}{a_0} \approx \left[\frac{(1-\nu)a_0 E}{8\gamma}\sqrt{\frac{\pi}{N}}\right]^{1/3}\left(\frac{u_{bk}}{b_k}\right)^{2/3}\left(\frac{b_k}{a_0}\right)^{4/3}. \tag{30}$$

Solving (29) with respect to l or using (30), it is possible to obtain the limiting dimensions of a set of N conical jointings arised in the medium as a result of an explosion.

CALCULATIONS OF BRITTLE MEDIUM BREAKING DYNAMICS IN CRUSHING AND RADIAL CRACK ZONES

Complete calculation of the development of breaking was accomplished as was shown above in two stages: in the first, development of the crushing zone was calculated up to the instant of emergence of the radial crack front into the depth of the medium, and in the second, the dynamics of the crack system was determined with a prescribed number of N jointings within it. The set of differential Eqs. (24) and (25) was solved numerically for unknown $a(t)$, $b(t)$, and $l(t)$.

Calculations were performed for the basic collection of parameters and with variation of them with the aim of revealing features of the process and ranking the model parameters with respect to the degree of their effect on the final medium breaking. Plots are given in Fig. 6 for the dependences $a(t)$, $b(t)$ (curves *1* and *2*) and $l(t)$ (curves *3*–*8*, when N = 4, 8, 16, 32, 64, 128) obtained with the basic collection of parameters. These data make it possible to determine for the calculated variant the maximum possible dimensions of the sets of N radial cracks.

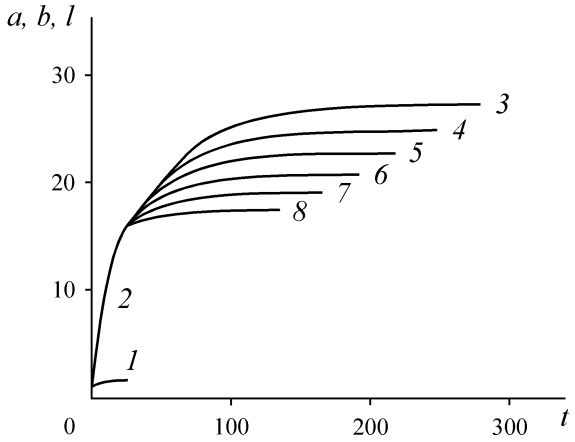

Fig. 6

Values of the parameters in the calculations were varied within the following ranges: $E = 5 \cdot 10^9 - 10^{11}$ Pa, $\rho = 1500 - 3500$ kg/m^3, $Y = 10^5 - 10^7$ Pa, $\alpha = 0.1 - 4$, $\gamma_0 = 10 - 80$ J/m^2, $v_{max} = 300 - 1300$ m/s, $P = 10^5 - 10^7$ Pa, $a_0 = 0.0225 - 0.1$ m, $P_0 = 10^9 - 10^{10}$ Pa, $\alpha_2 = 4 - 11.2$, $Y_2 = 5 \cdot 10^7 - 5 \cdot 10^8$ Pa, $\Lambda = -0.2 - 0.5$. This range includes the basic rocks (from coal to hard granites and diabases). A change in the parameters provided for one with respect to the basic collection gives for the final cavity radius, crushing wave, and the radial crack zone $(N = 8)$ the following estimates of the effect of different model parameters:

$$\frac{a_k}{a_0} \approx \frac{(3+\alpha_2)^{0.08} p_0^{0.14}}{\alpha^{0.16} E^{0.05} Y_2^{0.072} P^{0.02}}, \tag{31}$$

$$\frac{b_k}{a_0} \approx \frac{E^{0.34} (3+\alpha_2)^{0.58} p_0^{0.24}}{\alpha^{0.21} Y_2^{0.46} P^{0.07}}, \tag{32}$$

$$\frac{l_k}{a_0} \approx \frac{E^{0.021} p_0^{0.33} a_0^{0.26}}{\alpha^{0.27} Y_2^{0.006} (3+\alpha_2)^{0.05} P^{0.15} \gamma_0^{0.27} Y^{0.03} N^{0.13}}. \tag{33}$$

It follows from (31) that the final size of the cavity radius is more sensitive to changes in p_0 and α and it depends weakly on the rest of the problem parameters. According to (32) the final size of the crushing zone is mainly governed by the parameters α_2, Y_2, E, p_0 and α. The dependence of a_k and b_k on external pressure is quite weak. It follows from (33) that the defining parameters for the maximum radial crack zone radius l_k are p_0, a_0, α, γ_0, P and N.

Calculations of breaking parameters with different dilation coefficients Λ showed that its change has a marked effect on a_k and b_k, and particularly on l_k. The dependences of these values on Λ are close to linear for the main version of calculations (Fig. 7). It can be seen that with an increase in Λ from 0 to 0.5, l_k is almost doubled.

Quasi-static values of radial crack lengths l_{ks} were calculated from (29) $(l_{ks} = l)$ with $u_{bk} = u_{bm}$, $b_k = b_m$ determined from interpolation formulas (28) in terms of a_s, b_s, and u_{bs}. The results of calculations for $N = 8$ are given in Fig. 8 on coordinates l_k and l_{ks}, where l_k is the value of the final radius of the radial crack zone determined by solving the dynamic problem. Here, the following approximation relationship is fulfilled: $l_k = 0.96 l_{ks}$. It can be seen that formulas (29) make it

possible to calculate quite accurately l_k from known maximum values of b_m and u_{bm} obtained with calculation of the crushing zone development without a radial crack zone. This is a consequence of the fact that calculation of crack dynamics is carried out in a quasi-static approximation and that the values of b_k and u_{bk} are very close to b_m and u_{bm} within the range of change in parameters of the problem checked. A rougher estimate of l_{ks} by (30) gives a greater deviation from the dynamic solution, but with an accuracy up to 20% it may be used for analytical evaluation of the radial crack zone dimensions with explosion of a concentrated charge in monolithic rock. Formulas (29) and (30) make it possible to consider the effect of γ_0, N, and a_0 analytically if b_k and u_{bk} are known.

Results of calculations of set (23) – (25) with different α, Y_2, P and p_0 are provided in dimensionless form in Table 1 in order to illustrate the range of change in a_k, b_k, l_k, u_{bk}, and t_k. Here, t_k is total breaking time. The upper number in the boxes of Table 1 reflects the case of $\Lambda = 0$, and the lower reflects $\Lambda = 0.5$. In determining l_k it was assumed that $N = 8$ and $a_0 = 0.05$ m, and the rest of the parameters were taken from the basic collection. Values of $p_0 = 10^9$ and 10^{10} Pa correspond to weak and high explosive, and the values of $Y_2 = 5 \cdot 10^7$ and $5 \cdot 10^8$ Pa correspond to less hard and harder rock. Whence in rocks with low strength in compression explosive of a charge with a reduced initial pressure causes shear failure in the crushing zone without development of a radial crack zone. Conversely, in hard rock with use of high explosive the main breaking volume due to explosion of a concentrated charge applies to the crack zone.

We compare the data obtained with the results of experiments for explosive rock breaking. These tests are connected with considerable difficulties in determining the dimensions of the breaking zones due to confined explosions in rocks. Results are presented for studies [13] where a labor-intensive but very informative method was used for core drilling of the block being blasted before and after an explosion. Experimental explosions were carried out in quarries in the Malinsk granite deposit (Zhitomir region), in the Bulganak limestone quarry (Crimean region), and in concrete blocks (3×3×3.5 m) in an experimental base of the Ukrainian Academy of Sciences. The weight of the explosive charges was 120 – 200 g with a placing depth of 1.3 – 3 m. The dimensions of the intensive crushing zones obtained in concrete, granite, and limestone were 10 – 12.8, 11 – 14, and $8 - 12a_0$, and the radial cracks zones were 24, 26, and $20 - 24a_0$. These results are close to those in the case of $p_0 = 10^{10}$ Pa, $Y_2 = 5 \cdot 10^8$ Pa, $P = 10^5$ Pa corresponding to compact charging of explosive and a hard rock. More accurate verification of the adequacy of the theory and experiment could be provided with the existence of data for elastic and strength parameters of the rocks used in the experiment.

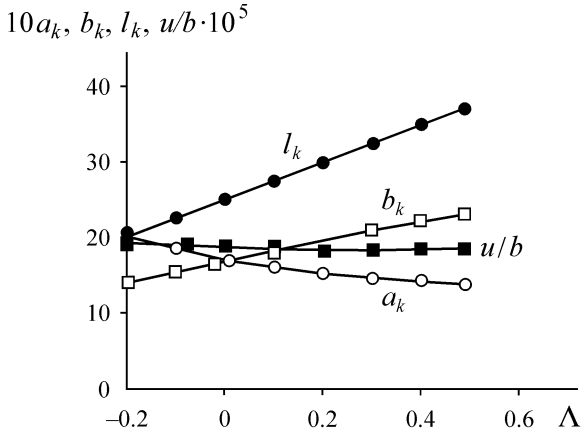

Fig. 7

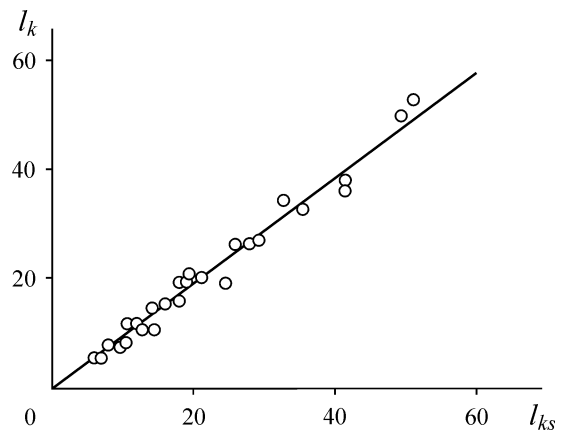

Fig. 8

TABLE 1

P, 10^5 Pa	$\alpha = 1.1$					$\alpha = 4$				
	a_k	b_k	l_k	$u_{bk} \cdot 10^3$	t_k	a_k	b_k	l_k	$u_{bk} \cdot 10^3$	t_k
	$p_0 = 1 \cdot 10^9$ Pa					$Y_2 = 5 \cdot 10^7$ Pa				
1	1.53	19.4	21	1.88	65	1.26	14.1	14.1	1.34	25
	1.27	33.6	41.4	3.2	129	1.1	19.3	19.3	1.82	32
100	1.31	10.8	10.8	2.0	18	1.14	7.95	7.95	1.42	12
	1.16	15.9	15.9	2.87	28	1.07	9.55	9.55	1.67	14
	$p_0 = 1 \cdot 10^9$ Pa					$Y_2 = 5 \cdot 10^8$ Pa				
1	1.2	5.55	17.9	5.8	80	1.1	4.34	12.6	4.46	60
	1.12	7.2	24.5	7.55	130	1.06	4.92	14.4	5.04	65
100	1.17	4.9	8.1	5.6	29	1.08	3.87	6.13	4.36	20
	1.1	6.18	10.2	7.1	34	1.05	4.3	6.7	4.8	20
	$p_0 = 3.3 \cdot 10^9$ Pa					$Y_2 = 1 \cdot 10^8$ Pa				
1	1.76	18.2	28	3.54	111	1.44	13.7	19.1	2.55	60
	1.42	29	49.5	5.48	192	1.22	18	29	3.58	124
100	1.55	12.6	12.6	3.44	19	1.32	9.8	9.8	2.58	14
	1.33	18.7	18.7	5.01	28	1.18	12.6	12.6	3.31	18
	$p_0 = 1 \cdot 10^{10}$ Pa					$Y_2 = 5 \cdot 10^7$ Pa				
1	2.45	33.2	41.4	3.26	126	1.82	22.9	25.6	2.16	62
	1.73	49.1	70.4	4.96	208	1.42	32.5	39.4	3.04	97
100	1.95	18.2	18.2	3.31	27	1.58	13.7	13.7	2.28	16
	1.55	25.8	25.8	4.5	35	1.35	18.2	18.2	2.99	22
	$p_0 = 1 \cdot 10^{10}$ Pa					$Y_2 = 5 \cdot 10^8$ Pa				
1	1.73	9.7	35.3	9.8	183	1.49	8.1	26.1	7.86	142
	1.48	13.6	51.2	13.4	240	1.32	10.2	34.8	9.91	169
100	1.66	8.86	14.5	9.7	43	1.45	7.45	11.6	7.82	33
	1.45	12.1	19.5	13	57	1.3	9.25	14.4	9.59	40

CONCLUSIONS

1. A fundamental stage for the whole process of brittle rock breaking by explosion of a concentrated charge is development of a shear crushing zone. The maximum explosion cavity and crushing wave radii obtained in this stage determine the intensity of crushing in the zone of radial cracks and their dimensions.

2. Ranking has been performed for medium and explosive parameters with respect to the degree of their effect on breaking. It has been shown that in order to develop a shear crushing zone the main parameters are the initial pressure in the detonation products, rock strength in compression, the friction angle in pulverized rock of the crushing zone, and the ratio of rock strength in uniaxial compression and tension.

3. In order to evaluate the maximum explosion cavity, crushing zone, and radial crack dimensions, approximate analytical relationships have been constructed that give a good enough approximation of data calculated by the dynamic model.

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