Journal of Mining Science, Vol. 42, No. 1, 2006

ROCK FAILURE

MODELING THE FAILURE OF ROCK BLOCKS BY BLASTING A CYLINDRICAL CHARGE

N. I. Aleksandrova and E. N. Sher UDC 622.235

The calculation model of breaking action exerted by a cylindrical charge blasted in a block rock mass is proposed. The deformation of a block medium in the zones of elasticity and heavy fragmentation is described by a continuum, properties of which are changed as compared with monolithic rocks. It is shown that the accounting of the block rock strength decrease in the massif influences greatly the calculated failure parameters and provides an opportunity to obtain fragmentation zone sizes, close to those observed in actual rock blasting.

Fractured rock, strength, explosion, deep-hole charge, fragmentation

The theoretical description of a destroying action exerted by blasting in a rock mass is often based on the zone approach [1, 2] for the confined concentrated charges. This study is the continuation of [3, 4], where the parameters of the deep-hole charge explosion in monolithic brittle rocks have been evaluated. In [3] the estimates were attained in a quasi-static approximation, neglecting the rock compressibility in the elastic zone and its loosening (dilation) in the zone of fragmentation. In [4] the dynamic behavior of the elastic zone and the dilation in the fragmentation zone are taken into account. The deformation of monolithic brittle rocks is featured by a fact that failure mostly occurs in the zones with radial cracks. In fractured block rocks with loosened interlayers, propagation of radial cracks with load-free edges is difficult. In this case it is reasonable to suppose that failure develops along fractures and interblock layers. Such fragmentation results in separating a rock mass into blocks in compliance with its hierarchic block structure. The analysis of the displacement fields in a rock mass after the large-scale blasting revealed that the deformation of the failed medium appears as translation and rotation of blocks [5, 6]. As a first approximation, it is possible to describe such deformation by the free-flowing medium model, where the slipping and friction of separate blocks, and the specific volume increase due to the dilation are taken into account.

In the present work, one of the research tasks deals with studying the mechanism of the failed block medium deformation. The parameters of this mechanism are assumed as free ones. They are varied in order to rank them and to identify the range of values within which the calculated sizes of failure zone are close to those obtained in real blasting operations.

Institute of Mining, Siberian Branch, Russian Academy of Sciences, E-mail: sher@misd.nsc.ru, Novosibirsk, Russia. Translated from Fiziko-Tekhnicheskie Problemy Razrabotki Poleznykh Iskopaemykh, No.1, pp. 31-38, January-February, 2006. Original article submitted November 9, 2005.

1062-7391/06/4201-0027 ©2006 Springer Science + Business Media, Inc.

PROBLEM STATEMENT

The problem is considered as an axisymmetric one (a plane strain state). In an infinite elastic medium, an explosive charge is placed in a cylindrical cavity of radius a_0. At the initial moment of blasting, gases with pressure p_0 fill the cavity. The gas pressure in the explosion cavity during its enlarging is calculated by the modified Jones – Miller adiabat for a TNT charge [7]:

$$p(a) = \begin{cases} p_0(a/a_0)^{-6}, & a \le a_*, \\ p_0(a_*/a_0)^{-6}(a/a_*)^{-2.54}, & a \ge a_*, \end{cases} \tag{1}$$

where $a_* = 1.89a_0$; $a = a(t)$ is the explosion cavity radius; t is the time. The initial pressure depends on the type of explosive. After blasting, the elastic and fragmentation waves propagate in the medium.

In the elasticity zone, the medium movement is described by the wave equation relative to the radial displacement u:

$$\frac{\partial^2 u}{\partial t^2} = c_p^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right), \tag{2}$$

where c_p is the velocity of longitudinal waves. The stress tensor components are found by the formulae:

$$\sigma_r = \rho c_p^2\left(\frac{\partial u}{\partial r} - \varepsilon\frac{u}{r}\right) - P, \quad \sigma_\theta = \rho c_p^2\left(-\varepsilon\frac{\partial u}{\partial r} + \frac{u}{r}\right) - P, \quad \varepsilon = -\frac{\nu}{1-\nu}, \tag{3}$$

where P is the rock pressure; ν is Poisson's ratio; ρ is the medium density; σ_r and σ_θ are the stress tensor components in the cylindrical coordinate system (r, θ).

It is supposed that the block medium failure in the elastic zone is determined by the limit failure curve in the plane $(\sigma_r, \sigma_\theta)$. These curves are plotted for a number of media with the help of biaxial tests of samples [8]. Approximately, their shape can be represented by a piecewise linear function (Fig. 1, solid lines), symmetric to the straight line $\sigma_r = \sigma_\theta$. Section *I* corresponds to failure at $\sigma_r > \sigma_\theta > 0$. Here, the failure criterion is $\sigma_\theta = -\sigma_t$. In section *II* the failure condition of Coulomb – Mohr's type is valid:

$$\sigma_\theta - \sigma_r\frac{\sigma_t}{\sigma_c} - \sigma_t = 0;$$

in section *III* it is of the Tresca type:

$$\frac{\sigma_\theta - \sigma_r}{2} = \tau_p,$$

where σ_t and σ_c are the uniaxial tensile and compressive strengths, respectively; τ_p is the limit shear stress. The values of τ_p for some rocks are cited in [8].

For the block media, σ_t^b and σ_c^b are substantially lower as compared with σ_t and σ_c. Thus, in [9] the data cited for a rock mass show the decrease in the compressive strength by a factor of $2-4$, in the tensile strength by a factor of $5-20$ if the rocks are microfractured and by a factor of $10-100$ if the rocks are macrofractured. The conditional strength curve for a block medium is given as the dotted line

in Fig. 1. It is quantitatively determined by σ_t^b, σ_c^b and by the inclination of section *II*. In the present paper, sections *II* of the strength curves for a block medium and a sample are supposed parallel, and section *III* of the strength curve for a block medium is neglected.

Thus, the conditions corresponding to different sections of the strength curve in Fig. 1 are fulfilled at the boundary between the elastic zone and the failure front at $r = b(t)$:

$$\begin{aligned} &\text{Section } I\text{: } \sigma_\theta = \sigma_t^b,\ \sigma_t^b = m\sigma_c^b/(1+\alpha_2),\ \alpha_2 = \sigma_c/\sigma_t - 1; \\ &\text{Section } II\text{: } (1+\alpha_2)\sigma_\theta - \sigma_r - Y_2 = 0,\ Y_2 = \sigma_c^b, \end{aligned} \tag{4}$$

where m is the coefficient of structural weakening. Using (3) for stresses, we obtain from (4) the failure conditions at $r = b(t)$:

$$\begin{aligned} &\text{Section } I\text{: } f_1 = \frac{u}{r} - \varepsilon\frac{\partial u}{\partial r} + P_1 = 0,\ P_1 = \frac{\sigma_t^b + P}{\rho c_p^2}; \\ &\text{Section } II\text{: } f_2 = A_2\frac{\partial u}{\partial r} - B_2\frac{u}{r} + P_2 = 0,\ P_2 = \frac{Y_2 + \alpha_2 P}{\rho c_p^2}; \end{aligned} \tag{5}$$

$$A_2 = (1+\alpha_2)\varepsilon + 1,\ B_2 = 1 + \varepsilon + \alpha_2.$$

In the fragmentation zone the medium movement is described by the one-dimensional equation:

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) = \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r}, \tag{6}$$

where v is the radial velocity.

It is supposed that in the fragmentation zone, the Coulomb – Mohr plasticity condition yields [7]:

$$\begin{aligned} &(1+\alpha)\sigma_\theta - \sigma_r - Y = 0; \\ &Y = \frac{2C\cos\varphi}{1-\sin\varphi},\ \alpha = \frac{2\sin\varphi}{1-\sin\varphi}, \end{aligned} \tag{7}$$

where C is the cohesion; φ is the internal friction angle.

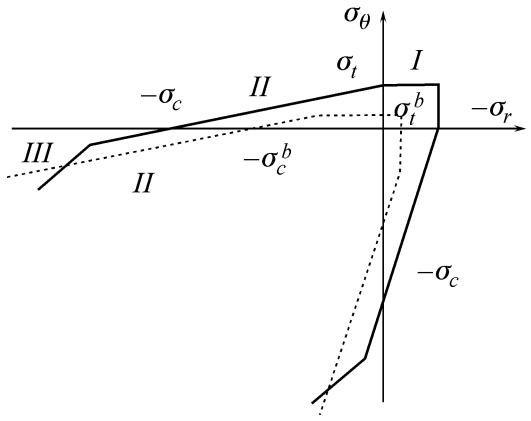

Fig. 1

The medium density ρ in (6) is determined by the material compression in the elasticity zone and subsequent compaction and loosening due to the dilation effect in the fragmentation zone. In the present studies, it is supposed that the effect of the medium density variations in the fragmentation zone in Eq. (6) is small and ρ is equal to the initial value. To describe the dilation effect, make an assumption on the relationship between the volumetric and shear strains, similarly to the assumption accepted in [2]:

$$\frac{\partial v}{\partial r} + \frac{v}{r} = \Lambda\left(\frac{v}{r} - \frac{\partial v}{\partial r}\right), \tag{8}$$

where Λ is the dilation coefficient. From (8) we have in the fragmentation zone:

$$v(r,t) = \dot{a}\left(\frac{a}{r}\right)^n, \quad n = \frac{1-\Lambda}{1+\Lambda}, \tag{9}$$

where $\dot{a}$ is the velocity of the explosion cavity radius variation.

On the boundary $r = a(t)$, the radial stress in the fragmentation zone depends on the gas pressure in the cavity:

$$\sigma_r(a+0) = -p(a). \tag{10}$$

On the boundary of the fragmentation zone $r = b(t)$, the stresses and velocities are supposed continuous:

$$\sigma_r(b-0) = \sigma_r(b+0), \tag{11}$$

$$\dot{u}_b = \dot{u}(b+0) = v(b-0) = \dot{a}\left(\frac{a}{b}\right)^n. \tag{12}$$

Find the general solution for the radial stress in the fragmentation zone from the equations (6), (8), and (9):

$$\sigma_r = \frac{Y}{\alpha} + \rho\left[\frac{(\dot{a}a^n)'_t}{g-n+1}r^{1-n} - \frac{n(\dot{a}a^n)^2}{g-2n}r^{-2n}\right] + \frac{F(t)}{r^g}, \quad g = \frac{\alpha}{1+\alpha}, \tag{13}$$

It depends on two unknown functions $a(t)$, $F(t)$.

Substituting the solution (13) into the boundary conditions (10) and (11), obtain the system of ordinary differential equations of the first order for $a(t)$ and $V(t) = \dot{a}(t)$:

$$\frac{da}{dt} = V, \quad \frac{dV}{dt} = \frac{p(a) - K_3 - V^2 n(K_1 - K_2)}{aK_1},$$

$$K_1 = \frac{\rho}{g+1-n}\left[\left(\frac{b}{a}\right)^{g+1-n} - 1\right], \quad K_2 = \frac{\rho}{g-2n}\left[\left(\frac{b}{a}\right)^{g-2n} - 1\right], \tag{14}$$

$$K_3 = \left(\frac{Y}{\alpha} - q\right)\left(\frac{b}{a}\right)^g - \frac{Y}{\alpha}, \quad q = \sigma_r(b+0).$$

Thus, the problem on deformation and failure of a block medium under blasting of a cylindrical charge is reduced to finding the functions $a(t)$, $b(t)$, and $u(t,r)$ in the elastic zone at $r > b(t)$, satisfying Eq. (2), failure condition (5), and system (14) at the following initial conditions:

$$t = 0, \quad a = b = a_0, \quad V = \frac{p_0 - P}{\rho\, c_p^2}, \quad u = 0, \quad \dot{u} = \delta(r - a_0)V,$$

where δ is the Dirac delta-function.

The numerical solution to the problem was executed by the following algorithm. For the wave equation (2) the explicit difference scheme of "cross" type was used. The grid steps of the scheme by the time τ and space h were chosen from Courant's stability conditions $c_p\tau \le h$. As the minimum of the numerical dispersion is reached at $c_p\tau = h$, so this condition is assumed in the calculations. The integration of the system of the ordinary differential equations of the first order, (14), was carried out by Euler's method. In the numerical calculations, the fragmentation zone radius is determined at each time step as the radius at which one of the equalities (5) is fulfilled.

At the initial stage, after blasting the cavity expands without medium failure till the moment t_1, when on the surface of the cavity the failure condition (5) is satisfied. Then, the movement parameters $a(t_1)$, $b(t_1)$, $V(t_1)$, and $u(t_1, r)$ are used as initial ones to solve the system (2) and (14) with the boundary conditions (5), (10) – (12).

The dynamic problem of the block medium failure by blasting a deep-hole charge is solved for the following values of the parameters: $E = 10$ GPa, $\rho = 2500\,\text{kg/m}^3$, $\nu = 0.3$, $\alpha_2 = 11.2$, $\alpha = 4$. In the calculations the strength parameters for block rocks, given no thorough study yet, were varied within the ranges: $Y = 20 - 200$ MPa, $Y_2 = 80 - 800$ MPa, $\Lambda = 0 - 0.5$, $m = 0.1 - 0.5$. The parameters of the deep-hole charges were assumed as: $a_0 = 0.0225$ m, $p_0 = 3.32$ GPa, rock pressure $P = 0.1$ MPa. At the dimensionless presentation of the parameters, a_0 is a unit of length, c_0 is velocity, a_0 / c_0 is time, E is stress.

Figure 2 shows the dependences of the fragmentation zone radii b on the dimensionless time, obtained at different dilations Λ and $Y = 40$ MPa, $Y_2 = 200$ MPa, $m = 0.2$. It is obvious that the dilation effect tells on the increase in the failure zone area and its formation time. The maximum sizes of the explosive charge cavity diminish with the growth of Λ.

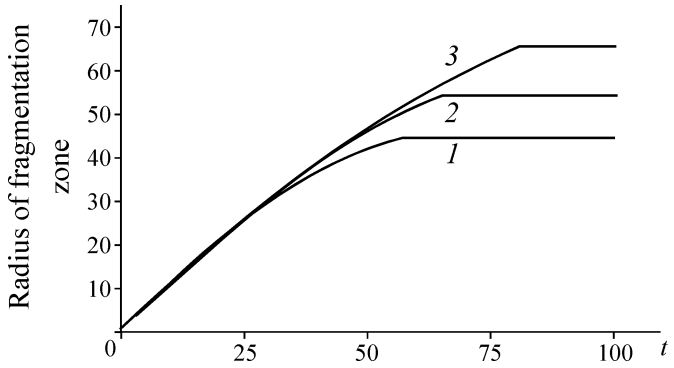

Fig. 2. The dilation effect on dynamics of the fragmentation zone development in time. Curves *1*, *2*, and *3* correspond to the dilation values Λ = 0, 0.25, and 0.5, respectively

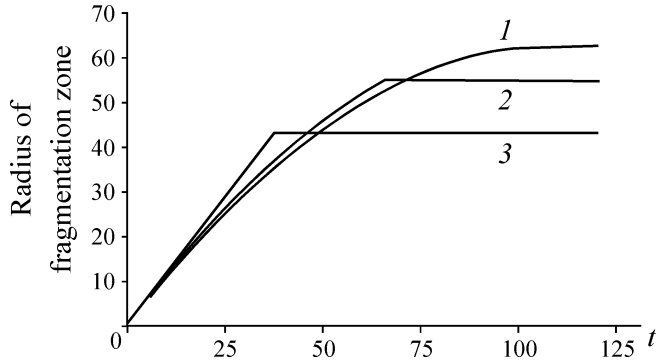

Fig. 3. The influence of cohesion in the fragmented rock under plastic deformation on the dynamics of the fragmentation zone development

The effect of the failed medium cohesion on the dynamics of the fragmentation zone development is shown in Fig. 3 (curves *1 – 3* correspond to $Y / Y_2 = 0.1, 0.2, 1$, respectively). Decreasing Y from 200 to 20 MPa results in the increasing radius of the failed zone by 50 %, the rest parameters in these calculations were $Y_2 = 200\,\text{MPa}$, $m = 0.2$, $\Lambda = 0.25$.

The effect of the structural weakening of a block medium, specified by the reduction in its tensile strength as compared with this parameter for a sample is demonstrated in Figs. 4 – 6 as the relationships between the maximum radius of the explosion cavity, failure zone, failure time, and value m (here $Y_2 = 200\,\text{MPa}$, $\Lambda = 0.25$ remained constant). The value of Y varied within the range of 20 – 200 MPa. From the relationships in Figs. 3 – 6, it follows that the structural rock mass weakening, caused by the block structure and fracturing, slightly affects the sizes of the explosion cavity and considerably increases the fragmentation zone area. The reduction in m by a factor of 5 provides the increase in the fragmentation zone radius by a factor of 3 and in the failure development duration by a factor of 1.5 – 2.

The compressive strength of a rock mass influences greatly the parameters of the deep-hole charge blasting in a block medium. In Fig. 7 the calculated dependences of the maximum radius of the explosion cavity, fragmentation zone, and failure time on the parameter Y_2, variable within the range $Y_2 = 80 - 800\,\text{MPa}$ at $m = 0.2$, $Y / Y_2 = 0.2$, are shown. From the curves in Fig. 7, it follows that the simultaneous lowering of Y_2 and σ_t^b by a factor of 10 contributes to the increase in the fragmentation zone radius by a factor of 4 and in the failure duration by a factor of 5.

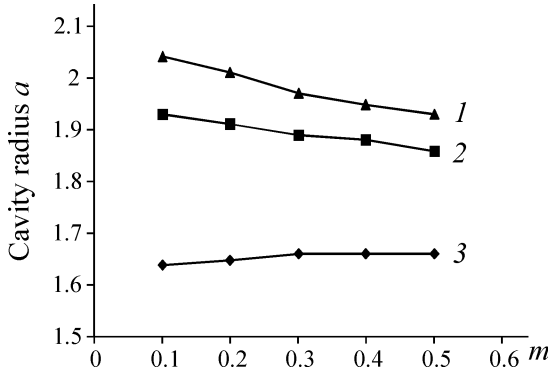

Fig. 4. Relationship between the explosion cavity radius a and the structural weakening coefficient m. Curves *1 – 3* correspond to the values $Y / Y_2 =$ 0.1, 0.2, 1, respectively

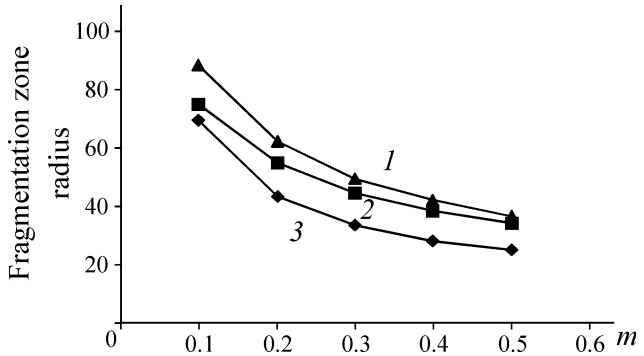

Fig. 5. Relationship between the failure zone radius b and m. The values of Y / Y_2 for curves *1 – 3* are given in Fig. 4

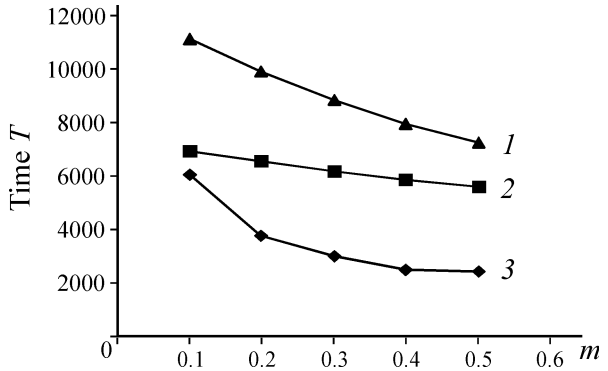

Fig. 6. Dependence of the failure zone formation time T on m. The values of Y / Y_2 for curves *1 – 3* are given in Fig. 4

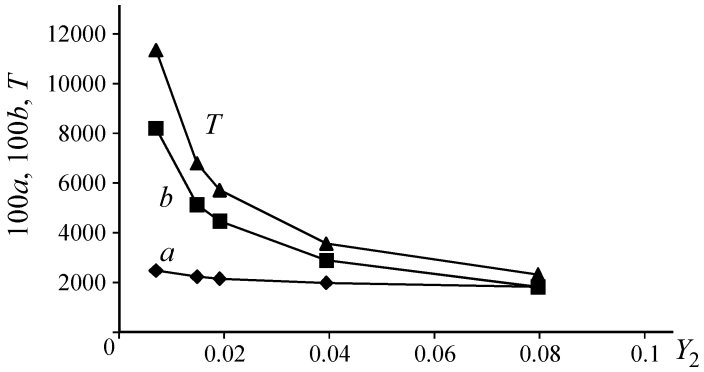

Fig. 7. Dependence of the explosion cavity radius a, failure zone b, and failure zone formation time T on the uniaxial compressive strength $\sigma_c^b = Y_2$

The calculated data obtained show that the parameters characterizing the structural weakening of a block mass such as the compressive σ_c^b and tensile σ_t^b strengths of a rock mass are the basic parameters affecting the value of the calculated fragmentation zone radius. In this connection, the in situ measurement of these parameters in a rock mass becomes the hottest problem. The calculations and the comparison of the results obtained with the data of blasting give the grounds to estimate the range of the mentioned parameters: the drilling-and-blasting borehole pattern depends on the rock type and

varies within $3-8$ m at a borehole diameter of 250 mm. Under assumption that the diameter of the fragmentation zone resulted from blasting one hole is 1.5 spacing of the borehole pattern, the fragmentation zone radius is 15 – 40 in the dimensionless form. These values are obtained in the calculation model at $\sigma_c^b = 160-800$ MPa, $\sigma_t^b = 3.3-33$ MPa.

The study was conducted with financial support from the Russian Foundation for Basic Research, Project No. 05-05-64837.

REFERENCES

1. S. S. Grigoryan, "Some aspects of mathematical theory of deformation and failure of hard rocks," *Prikl. Mat. Mekh*., **31**, Issue 4 (1967).
2. V. N. Rodionov, V. V. Adushkin, V. N. Kostyuchenko, V. N. Nikolaevskii, A. N. Romanov, and V. M. Tsvetkov, *Mechanical Effect of Underground Blasting* [in Russian], M. A. Sadovsky (ed.), Nedra, Moscow (1971).
3. E. N. Sher, "Consideration of dynamics in description of friable rock failure by string charge blasting," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 3 (1997).
4. N. I. Aleksandrova and E. N. Sher, "Effect of dilation of rock breaking by explosion of a cylindrical charge," *Fiz.-Tekh. Probl. Razrab. Polezn. Iskop*., No. 4 (1999).
5. M. V. Kurlenya, V. V. Adushkin, V. N. Oparin, et al., "Alternating response of rocks to a dynamic impact," *Dokl. AN SSSR*, **323**, No. 2 (1992).
6. M. V. Kurlenya, V. N. Oparin, A. F. Revuzhenko, and E. I. Shemyakin, " On some specific features of rock response to blasting impact in the close vicinity," *Dokl. AN SSSR*, **293**, No. 1 (1987).
7. G. P. Chadwick, A. D. Cox, and H. G. Hopkins, "Mechanics of deep underground explosions," in: *Philosophical Trans. of the Royal Society of London, Ser. A, Mathematical and Physical Sciences*, **256**, No. 1070 (1964).
8. A. N. Stavrogin and V. S. Georgievskii, *Catalogue of Mechanical Properties of Rocks* [in Russian], VNIMI, Leningrad (1968).
9. V. V. Rzhevskii and G. Ya. Novik, *Fundamentals of Rock Physics* [in Russian], Nedra, Moscow (1984).