Home Test 1

1. Does there exist a permutation x satisfying the following conditions:
(a) $x \in S_9$,
(b) $\mathrm{sgn}(x) = 1$,
(c) $|x| = 10$?

2. Let $a = (1, 2, \dots, n) \in S_n$ be the cycle of length n. Solve in S_n a "quadratic" equation: $x^2 = a$.

3. Let G be a finite abelian permutation group. Prove that
(a) if G is transitive, then it is regular;
(b) if G is primitive, then G has a prime order.

4. Let H be a group. Set $G = H \times H$ and $\Omega = H$. Prove that
(a) the rule: $x^{(g,h)} = g^{-1}xh$ for all $x \in \Omega$ and $(g, h) \in G$, defines an action of G on Ω.
(b) the action is always transitive (find the stabilizer G_e of the neutral element $e \in H = \Omega$);
(c) the action is faithful (has the trivial kernel) $\Longleftrightarrow$ the center $Z(H) = 1$;
(d) the action is primitive $\Longleftrightarrow$ H is a simple group.

5. For the graph Γ from Figure 2.1 (a) on Page 39 of [DM],
(a) find the order of $\mathrm{Aut}(\Gamma)$;
(b) find some set of generators of $\mathrm{Aut}(\Gamma)$;
(c) prove that $\mathrm{Aut}(\Gamma)$ is transitive and find a point stabilizer;
(d) prove that $\mathrm{Aut}(\Gamma)$ is imprimitive and find all nontrivial blocks.

6. (a) Find the group of rotations of regular tetrahedron.
(b) How many fundamentally different tetrahedrons with faces painted in 3 colors are there?