Home Test 1 new

1. Solve in S_{10} the equation: $(1,3)(5,9)(5,7)(3,9)\cdot x^2=(2,4,6,8,10)$.

2. For which n, can the cycle $(1,2,\dots,n)$ be represented as a product of cycles of length 3? Find such a representation whenever possible.

3. Let $G\le \mathrm{Sym}(n)$. Prove that
(a) for $\alpha\in\Omega$, the subset $\Delta=\{\beta\in\Omega|(\alpha,\beta)\in G\}\cup\{\alpha\}$ of Ω is a block;
(b) if G is primitive and contains a transposition, then $G=\mathrm{Sym}(n)$.

4. Let H be a normal subgroup of $G\le\mathrm{Sym}(n)$. Prove that
(a) if G is 2-transitive, then G is primitive;
(b) if G is primitive, then H is transitive;
(c) if G is 2-transitive, then H is $\frac{3}{2}$-transitive, that is H is transitive and the orbits (distinct from $\{\alpha\}$) of a point stabilizer H_α are of the same size.

5. A regular triangular prism $\mathcal{P}$ is a triangular prism whose bases have regular triangles and the side faces are rectangles.
(a) Find the order and some generating set of the group of symmetries of $\mathcal{P}$.
(b) Find the order and some generating set of the group of rotations of $\mathcal{P}$.
(c) How many fundamentally different prisms $\mathcal{P}$ with faces painted in 3 colors are there?

6. For the graph Γ from the figure below,
(a) find the order of $\mathrm{Aut}(\Gamma)$;
(b) find some set of generators of $\mathrm{Aut}(\Gamma)$;
(c) prove that $\mathrm{Aut}(\Gamma)$ is transitive and find a point stabilizer;
(d) prove that $\mathrm{Aut}(\Gamma)$ is primitive.