1. Let H be a subgroup of G. Prove the following statements:

   (a) $HgHg \subseteq Hg$ if and only if $g \in H$;

   (b) if $xH = Hy$, then $yH = xH = Hx = Hy$.

2. Prove that A_4 does not possess subgroups of order 6.

3. Each group of even order contains an element of order 2.

4. Prove that $\mathrm{Aut}(\mathbb{Z}_7)$ is cyclic, while $\mathrm{Aut}(\mathbb{Z}_{15})$ is not, where $\mathbb{Z}_n$ denotes the cyclic group of order n.