may show that

$$E \leqslant \frac{1}{n}\left(\frac{9}{8}+\frac{1}{2}\tau^2+\tau^4\right)e^{-\tau^2}. \tag{45}$$

Since $\tau^2(r+1)^2 \geqslant \frac{(r+1)^2}{2}\tau^2+\tau^2 \geqslant \frac{n}{2}+\tau^2$ for $r \geqslant 1$, then from the first inequality in (41) it follows that

$$|I_2| \leqslant 2e^{-n/2-\tau^2} \leqslant \frac{4}{ne}e^{-\tau^2}. \tag{46}$$

Substituting (45) and (46) into (44), we obtain assertion (7). The theorem is proved.

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THREE-DIMENSIONAL HYPERBOLIC MANIFOLDS OF LÖBELL TYPE

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1. Answering in the affirmative a question of Koebe on the existence of spatial Clifford–Klein forms of constant negative curvature, Löbell [1] constructed the first example of a closed orientable three-dimensional hyperbolic manifold. Other examples are contained in [2-5], for example.

The first examples of closed nonorientable hyperbolic manifolds appeared in [6, 7].

The paper [1], based on complicated geometrical constructions, remained unnoticed for a long time. Carrying over Löbell's idea to algebraic language, as we do in the present paper, enables us to construct an infinite series of closed three-dimensional hyperbolic manifolds, both orientable and nonorientable (Theorems 1 and 2). From this it follows that the first example of such a nonorientable manifold is essentially due to Löbell.

2. Let R be a polyhedron with right dihedral angles in the Lobachevskii space H^3, and G the group generated by reflections in its faces. In order to show how to glue the manifold from eight copies of R, we consider an epimorphism into the 8-element group $\varphi: G \to Z_2^3$. We observe that the group Z_2^3 can be regarded as a vector space over the field GF(2). Arguments close to those in [6] and based on the fact that the stabilizer in G of each vertex of R is isomorphic to Z_2^3 enable us to establish the following assertion.

LEMMA 1. The kernel $\operatorname{Ker}\varphi$ of the epimorphism $\varphi: G \to Z_2^3$ does not contain elements of finite order if and only if the images of the reflections in any three faces of R that have a common vertex are linearly independent in the group Z_2^3, regarded as a vector space.

Later, in the construction of orientable manifolds, we shall use the following result.

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LEMMA 2. If G is generated by reflections in faces of a polyhedron with right dihedral angles, and φ: $G\to \mathbf{Z}$ is an epimorphism that takes the generators of G into four elements of $\mathbf{Z}_2^3$, any three of which are linearly independent in $\mathbf{Z}_2^3$ regarded as a vector space, then $\operatorname{Ker}\varphi$ does not contain elements that change the orientation.

Proof. Four elements of $\mathbf{Z}_2^3$ any three of which are linearly independent in $\mathbf{Z}_2^3$, regarded as a vector space over GF(2), are α, β, γ, δ, where α, β, γ are any three linearly independent elements, and $\delta = \alpha + \beta + \gamma$. Since G is generated by reflections, the elements in it that change the orientation are words of odd length in the generators of G. Consequently, the image of such an element under the epimorphism is a word of odd length in α, β, γ, δ, and since $\delta = \alpha + \beta + \gamma$, also of odd length in α, β, γ. But α, β, γ are linearly independent. Thus the elements that change the orientation are not in $\operatorname{Ker}\varphi$.

3. For the construction of manifolds we consider polyhedra of the following type.

Let ABCA'B'C' be a triangular prism. In it we draw the edge DE with vertices D and E lying in BB' and CC', respectively. By a theorem of Andreev [8], for any integer $n \geq 5$ in the Lobachevskii space H^3 there is a convex bounded hexahedron ABCA'B'C'DE with dihedral angles π/n, $\pi/4$, $\pi/4$ at the edges AA', BD, EC', respectively, and with right angles at the other edges.

Let $\Delta(n)$ be the group generated by reflections in the faces of this hexahedron. The elements of $\Delta(n)$ that leave the edge AA' fixed form the dihedral group D_n of order 2n. Under the action of D_n, from 2n copies of the hexahedron there is formed a (2n + 2)-hedron R(n) whose lateral surface consists of two regular right-angled n-gons and 2n right-angled pentagons.

We note that R(5) is a regular dodecahedron with right dihedral angles, while R(6) was first constructed in [1].

Let G(n) denote the group generated by reflections in the faces of R(n). It has the following presentation:

generators:

$$g_1, g_2, \ldots, g_{2n+2},$$

relations:

$$g_i^2 = 1, \quad i = 1, \ldots, 2n+2,$$

$$g_i g_{2n+1} = g_{2n+1} g_i, \quad g_{n+i} g_{2n+2} = g_{2n+2} g_{n+i}, \quad i = 1, \ldots, n,$$

$$g_1 g_n = g_n g_1, \qquad g_{n+1} g_{2n} = g_{2n} g_{n+1},$$

$$g_i g_{n+i+1} = g_{n+i+1} g_i, \quad i = 1, \ldots, n-1,$$

$$g_i g_{n+i} = g_{n+i} g_i, \quad i = 1, \ldots, n,$$

$$g_i g_{i+1} = g_{i+1} g_i, \quad i = 1, \ldots, 2n-1.$$

Consider the epimorphism φ_n: $G(n)\to \mathbf{Z}_2^3$ whose kernel $\Gamma_n = \operatorname{Ker}\varphi_n$ does not contain elements of finite order.

Definition. A three-dimensional hyperbolic manifold $L(n) = H^3/\Gamma_n$, where Γ_n is as described above, is called a manifold of Löbell type. If, in addition, $\varphi_n(g_{2n+1}) = \varphi_n(g_{2n+2})$, then L(n) is called a standard manifold of Löbell type.

We observe that L(n) is not determined uniquely by n and Löbell [1] constructed a standard orientable manifold of Löbell type for n = 6, while Al-Jubouri [6] constructed a nonstandard nonorientable manifold of Löbell type for n = 5.

4. We present the main results concerning the existence of manifolds of Löbell type.

THEOREM 1. For any integer $n \geq 5$ there is an orientable manifold of Löbell type L(n).

Proof. For any $n \geq 5$ we specify an epimorphism φ_n: $G(n)\to \mathbf{Z}_2^3$ that we need in the definition of a manifold of Löbell type. Suppose that α, β, γ are linearly independent in $\mathbf{Z}_2^3$, and that $\delta = \alpha + \beta + \gamma$. We put

$$\varphi_n(g_{2n+1}) = \varphi_n(g_{n+2i-1}) = \alpha, \quad \varphi_n(g_{2i-1}) = \beta,$$

$$\varphi_n(g_{2n+2}) = \varphi_n(g_{2i}) = \gamma, \quad \varphi_n(g_{n+2i}) = \delta,$$

$$i = 1, \ldots, k,$$

if n = 2k is even, k ≥ 3, and

$$\varphi_n(g_{2n+1})=\varphi_n(g_{n+2i-1})=\alpha,\quad \varphi_n(g_{2i-1})=\varphi_n(g_{2n})=\beta,$$
$$\varphi_n(g_{2n+2})=\varphi_n(g_{2i})=\gamma,\quad \varphi_n(g_{n+2i})=\varphi_n(g_n)=\delta,$$
$$i=1,\ldots,k,$$

if n = 2k + 1 is odd, k ≥ 2.

It is not difficult to see that the conditions of Lemmas 1 and 2 are satisfied for an epimorphism φ_n specified in this way. Consequently, $\Gamma_n=\operatorname{Ker}\varphi_n$ does not contain elements of finite order or elements that change the orientation. Thus for any n ≥ 5 we have an epimorphism φ_n that specifies an orientable manifold of Löbell type L(n).

LEMMA 3. A standard orientable manifold of Löbell type L(n) exists if and only if n = 3k, k ≥ 2. It is unique for every k.

Proof. To obtain the required manifold we need to impose the following conditions on the epimorphism $\varphi_n\colon G(n)\to \mathbf{Z}_2^3$:

(1) $\varphi_n(g_{2n+1})=\varphi_n(g_{2n+2})$;

(2) the images of the reflections in any three faces of R(n) that have a common vertex are linearly independent;

(3) $\operatorname{Ker}\varphi_n$ does not contain elements that change the orientation.

Let α, β, γ be linearly independent in Z_2^3. Without loss of generality we may assume that

$$\varphi_n(g_{2n+1})=\varphi_n(g_{2n+2})=\alpha,\quad \varphi_n(g_1)=\beta,\quad \varphi_n(g_2)=\gamma.$$

Then from (2) and (3) it follows that $\varphi_n(g_{n+1})=\delta=\alpha+\beta+\gamma$. Similarly, if $\varphi_n(g_2)=\gamma$, $\varphi_n(g_{n+1})=\delta$, then $\varphi_n(g_{n+2})=\beta$. By induction we obtain

$$\varphi_n(g_i)=\varphi_n(g_{n+i+1})=\beta,\quad i\equiv 1\pmod 3,$$
$$\varphi_n(g_i)=\varphi_n(g_{n+i-2})=\gamma,\quad i\equiv 2\pmod 3,$$
$$\varphi_n(g_i)=\varphi_n(g_{n+i-2})=\delta,\quad i\equiv 0\pmod 3.$$

Since to satisfy (2) reflections in adjacent faces must be mapped into different elements, n must be a multiple of 3. If n = 3k, k ≥ 2, then the epimorphism

$$\varphi_n(g_{2n+1})=\varphi_n(g_{2n+2})=\alpha,\quad \varphi_n(g_{3i-2})=\varphi_n(g_{n+3i})=\beta,$$
$$\varphi_n(g_{3i-1})=\varphi_n(g_{n+3i-2})=\gamma,\quad \varphi_n(g_{3i})=\varphi_n(g_{n+3i-1})=\delta,$$
$$i=1,\ldots,k,$$

specifies a standard orientable manifold of Löbell type. Since at each step of the construction of φ_n the image of the reflection in the next face is determined uniquely, for every k ≥ 2 the standard orientable manifold of Löbell type L(3k) is unique up to a change of basis in $\mathbf{Z}_2^3$, regarded as a vector space over GF(2).

From Lemma 2, Theorem 1 and Lemma 3 there follows immediately an assertion that estimates the number of manifolds of Löbell type for small values of n.

COROLLARY. The number of orientable manifolds of Löbell type L(n) for n = 5, 6, 7 is equal to 1, 4, and 3, respectively.

THEOREM 2. For any integer n ≥ 5 there is a nonorientable manifold of Löbell type L(n).

Proof. To obtain the necessary manifold we require that the kernel $H_n = \operatorname{Ker}\psi_n$ of the epimorphism $\psi_n\colon G(n)\to \mathbf{Z}_2^3$ should have no elements of finite order but should contain elements that change the orientation. We put

$$\psi_n(g_{2n+1})=\psi_n(g_{2n+2})=\alpha,$$
$$\psi_n(g_j)=\varphi_n(g_j),\quad \psi_n(g_{n+j})=\varphi_n(g_j)+\alpha,\quad j=1,\ldots,n,$$

where φ_n is the epimorphism described in Theorem 1. From the explicit form of the epimorphism it is obvious that the condition of Lemma 1 is satisfied. Also, elements of the form $h_j = g_{2n+1}g_jg_{n+j}$, where $1\leqslant j\leqslant n$, that change the orientation lie in H_n.

Thus the epimorphism ψ_n for any integer n ≥ 5 specifies a nonorientable manifold of Löbell type.

Remark. The theorem we have proved confirms the assertion of Löbell [1] that from eight copies of R(6) we can obtain by a suitable gluing both an orientable and a nonorientable manifold.

Thus the first example of a closed nonorientable three-dimensional hyperbolic manifold is essentially due to Löbell.

We note that by Lemma 2 a positive solution of the four-color problem implies the following: from eight copies of any manifold in H^3 with right dihedral angles we can glue a closed orientable three-dimensional hyperbolic manifold.

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GENERALIZED GROTHENDIECK CATEGORY

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The category Pro-Ab has been constructed in [1]; it is the category of spectra over all small categories which can be viewed as a full subcategory of the category $\mathrm{Funk}^{op}(\mathrm{Ab}, \mathrm{Ab})$. The purpose of this article is to show that Pro-Ab is an Abelian category and, moreover, the kernel and the cokernel of a morphism in the category Pro-Ab coincide with the kernel and the cokernel of this morphism in the category $\mathrm{Funk}^{op}(\mathrm{Ab}, \mathrm{Ab})$ which is known to be Abelian.

Definition 1. A spectrum over a small category I in the category Ab of Abelian groups is any functor $F\colon I \to \mathrm{Ab}$.

We will define a spectrum by specifying a set $A = \mathrm{Ob}\, I$, a family $\{X_a\}_{a\in A}$, $X_a = F(a)$, and, for each pair $a_1, a_2 \in A$, a family

$$p(a_1, a_2) = \left\{F\left(q_{a_1}^{a_2}\right)\right\}_{q_{a_1}^{a_2} \in \mathrm{Hom}_I(a_1, a_2)}.$$

The notation $\mathbf{X} = \{X_a, P(a_1, a_2), A\}$ is henceforth fixed for spectra.

We define the category Pro-Ab: its objects are all spectra over small categories. The morphisms are defined as follows: each spectrum $\mathbf{X} = (X_a\ P(a_1, a_2), A)$ determines a functor X^*: Ab → Ab by the formula $X^* = \lim\limits_{\to J} h^{X_a}$, where $h^{X_a} = Ab(X_a, -)$ is a corepresentable functor. The formula

$$\text{Pro-Ab}(\mathbf{X}, \mathbf{Y}) = \mathrm{Funk}^{op}(\mathbf{X}^*, Y^*) \tag{1}$$

determines the set of morphisms from the object X to the object Y. The composition of morphisms is defined as the composition of natural transformations of functors. Obvious equalities allow us to transform the expression (1):

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