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Efficient band monitoring with sensors outer positioning

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The article considers models for monitoring a band with a preset width using sensor networks shaped as disc covers. Every cover disc is a centred sensor operation area. The researchers determine a min-density band cover with the discs of one, two and three radii. The specific requirement for the cover is that the disc centres shall not be inside the band (external monitoring). Various efficient cover models are proposed and their characteristics are determined.

Keywords: disc covers; cover density; wireless sensor networks

AMS Subject Classifications: 52C15; 90C27; 51M15

1. Introduction

Wireless sensor network (WSN) consists of sensors that are placed in the monitoring area and use wireless communication to exchange information. The main sensor functions are to collect data (sensing), primary processing and transmission of collected data. In this case, the sensing area of each sensor is usually represented as a disk with a certain radius centred at the location of the sensor, and it is said that the sensor covers this disk. The mail issue in WSNs is to save the energy of the sensors.[1–3,7,11–13]

A cover of plane region S is such a set of disks C, where each point of the region belongs to at least one of disk. The cover density is defined as the ratio of the area of all disks in C to the area of S. Under the regular cover means a cover of the plane region by disks in which the whole region can be divided into regular polygons (tiles), forming a regular lattice. In this case, all the polygons should be covered equally. Obviously, regular covering density is the same for each tile, and coincides with coverage density of whole region.

Since the sensing energy consumption is proportional to the covered area, the main task of the WSN – lifetime maximization problem – leads to the problem of constructing minimum density cover. In the literature, a number of coverage models are proposed that use disks of different radii.[1,3,11,12] The values of the radii are chosen by solving the problems of minimizing the cover density. The density is a function (often multi-criteria) of several variables, and the search for the minimum is a difficult constrained optimization problem, which cannot always be solved analytically.

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The cover effectiveness is characterized by the total overlapping of disks. Most of the known cover models assume that the sensing ranges R of all sensors are the same. In this case, the minimum sensing energy consumption is achieved when each triple of neighbouring sensors forms an equilateral triangle with sides $R\sqrt{3}$.[8] The emergence of sensors with adjustable sensing ranges provides additional opportunities to improve the efficiency of the WSN.[1–3,7,11]

The models of regular plane covers using various discs and their placement structures are considered in [1–5,12]. Among the cover advantages is not only its small density but constructiveness as well: a simple structure of a cover grid and insignificant difference among different discs dimensions. So, in [3], comparative characteristics of the models with triangular and square structures of disc positioning are investigated. Contribution [4] determines minimal density $d \approx 1.018955$ of the cover using discs of two types without the limitation for the correlation between the disc dimensions. Note that we have managed to obtain the exact analytical value of this indicator:

$$d = \frac{2\pi}{\sqrt{27}}\left(1 - \sqrt{3}\cdot tg\left(\frac{\pi}{6} - \frac{\sqrt{27}}{12}\right)\right) \approx 1.018955892.$$

The proof of this result is planned for publication in a separate paper for the nearest future.

Contribution [5] determines the lower boundaries of the cover densities at preset correlation $q \geq 1$ between the area of large and small discs participating in the plane coverage. The result specified [4] implies that value q is rather large. Such model is useless for practical application. When building a sensor net, index q should be as close to one as possible.

In a sense, the problem of min-density cover determination is more complex and interesting compared to the problem of packing,[9,10,14] since the adjacent discs may overlap to a variable extent. Besides, the problems of covers are closely related to the applications for various areas monitoring by sensor networks that is, at present, rather topical.[1–3,7,11–13] During monitoring of a real object, we are to take into account its shape area and boundary extension. A result was obtained by Johnson et al. [6], stating that the number of unit discs required to cover any simple bound zone with area a and perimeter p equals to

$$n = \left[\frac{2a}{\sqrt{27}} + \frac{2p}{\pi\sqrt{3}} + 1\right].$$

In [7], there are models of min-density band covers with different radii strictly considering boundary effects that are convenient for applications. In this paper, we perform similar investigations, however, requiring external monitoring of the area. This means that, due to some inaccessibility, we cannot place sensors inside the controlled area.

The paper is structured as follows. Section 2 contains preliminary results related to general properties of external covers and their connections with plane covers. Section 3 considers two optimal coverage variants by discs of one radius with an equal density, but a different structure. Several models of covers using discs of two and three radii, respectively, are studied in sections 4 and 5. The conclusion includes the summary table with calculations results and brief analysis of the considered coverage models.

2. Preliminary results

Let a rectilinear band SP with width h be covered with discs,

$$SP \subseteq \bigcup_{i \in I} S_i,$$

where I denotes a countable set of numbers and $\{S_i\}$ is a set of discs of different radii. We consider only regular disc covers determined as follows. Let the band be divided into equal fragments F_j, which are commonly called as tiles.

$$SP = \bigcup_{j \in J} F_j, \quad \mu\left(F_{j_1} \bigcap F_{j_2}\right) = 0 \quad \forall j_1,\ j_2 \in J,$$

where μ is a measure of the area. In other words, the adjacent tiles may have only common boundary sections. Marked fragment F is covered with a finite set of discs, and all other fragments are covered equally with the same set of discs that is formally determined with the isometric transformation of plane T_j:

$$F \subseteq \bigcup_{m=1}^{k} S_m, \quad F_j = T_j(F), \quad F_j \subseteq T_j\left(\bigcup_{m=1}^{k} S_m\right), \quad j \in J. \tag{1}$$

For a regular band cover, the density is determined correctly:

$$D = \frac{\mu\left(\bigcup_{m=1}^{k} S_m\right)}{\mu(F)}.$$

The band is covered externally if the cover disc centres lie beyond the band or at its boundary. During investigations of specific models, the limitations for a number of different disc dimensions and specificity of these centres positioning in the typical fragment are agreed in addition. The following optimization problem is formulated in a general statement:

Problem For the given rectilinear band SP with width h and some suitable typical fragment F, meeting the condition (1), find a regular outer band disc cover with minimal density D at some known limitations for the system of cover discs.

For the solution of the problem on the outer band coverage, the following obvious statements are true.

LEMMA 1 *To search for the outer band cover with a minimal density, we need condition L1: the disc centres should be located at the band boundary. Otherwise, the density of the cover may always be reduced.*

LEMMA 2 *The regular outer band coverage with condition L1 determines the regular cover of the whole plane, which is obtained by multiple reflection of the original band cover in relation to its boundaries. At that, the band cover density is twice larger than the respective plane cover density.*

Let us explain the last statement with the help of Figure 1. Each disc 'participates' in the outer band coverage almost by its half. Full discs are used for the respective plane cover.

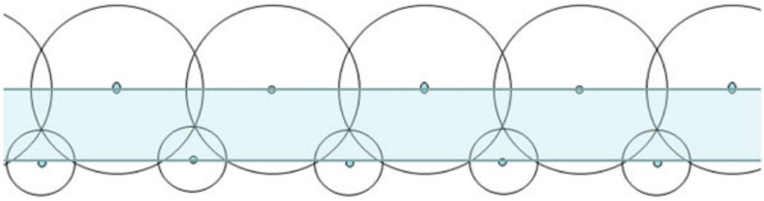

Figure 1. Example of regular outer cover of a band.

From Lemma 2, it may be inferred that all effective outer band covers for every individual class may be found from the respective optimal plane cover. At that, the known density indicators and important cover parameters may be used.

On the other hand, not every optimal plane cover of a certain type produces a ‘good’ band cover. The plane cover with a complex positioning of discs with different radii does not allow one to ‘cut out’ a band so that all discs covering it participate in a cover by their halves.

Let us use the following designations for different classes of covers.

(1) General designation: $\mathrm{Cov_F}(n,\ k)$, where n is the number of discs applied in a cover of a typical fragment F; k – a number of different disc’s types participating in a coverage.
(2) Structural designation: $\mathrm{Cov_F}(n\colon\ k_1/p_1, k_2/p_2, \dots, k_n/p_n)$, where k_i is a number of the i kind discs participating in the coverage of a typical fragment F, p_i – parts of discs of type i covering fragment F.

It is convenient to assume that fragment F is minimal; that is, it cannot be split into parts, which can also be considered as fragments. For example, for the cover, shown in Figure 1, a minimal fragment is the right-angled triangle with two equal sides (Figure 2), and the designations of this cover will be as follows: $\mathrm{Cov_F}(2,\ 2)$ and $\mathrm{Cov_F}(2\colon\ 1_8, 1_8)$. Note that a sign ‘/’ in the second designation may be removed. Both discs covering the fragment participate in its coverage by their eighths. Other parts of these discs cover similar fragments that are not shown in the figure.

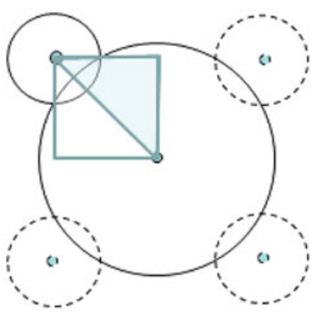

Figure 2. Minimal cover $\mathrm{Cov_F}$ $(2\colon\ 1_8, 1_8)$ fragment.

It is also important that, at the minimal fragment, the designations of the band cover and the respective plane always coincide.

3. Band coverage with discs of one radius

Let us consider the optimal (min-density) plane coverage by the discs of one radius R having a regular grid of disc positioning.[8] It is easy to notice that it generates two optimal band covers SA-1(1) and SA-1(2) with the same densities but different structures (Figure 3).

The main characteristics of the models are:

Model SA-1(1): density $D_{SA-1(1)} = 2D_{A-1} = 4\pi/\sqrt{27} \approx 2.4184$, when $R = 2\,h/3$.
Model SA-1(2): density $D_{SA-1(2)} = 2D_{A-1} = 4\pi/\sqrt{27} \approx 2.4184$, when $R = 2\,h$.

Note that these results might have been obtained during consideration of the elementary fragments of a cover that determine two of the mentioned coverage options at a different positioning in a band. Below, we demonstrate this method for more complicated models.

4. Band coverage with discs of two radii

Covering the plane with the discs of two radii 'capital' R and 'small' r, we have two structural models: Model A-2 and Model B-2 (Figure 4). Investigation of these models is performed in [5]; and we provide their main parameters.

Model A-2: density $D_{A-2} = \frac{11\pi}{18\sqrt{3}} \approx 1.1084$, when $r = \frac{R}{\sqrt{31}} \approx 0.1796R$.

Model B-2: density $D_{B-2} = \frac{3\pi}{8} \approx 1.1781$, when $r = \frac{R}{\sqrt{5}} \approx 0.4472R$.

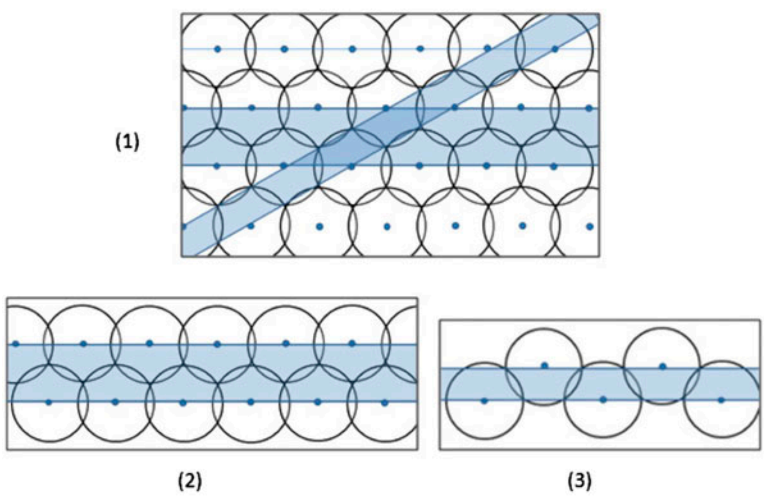

Figure 3. (1) Model A-1 of plane cover with discs of one radius and the respective models of band coverage: (2) Model SA-1(1) and (3) Model SA-1(2).

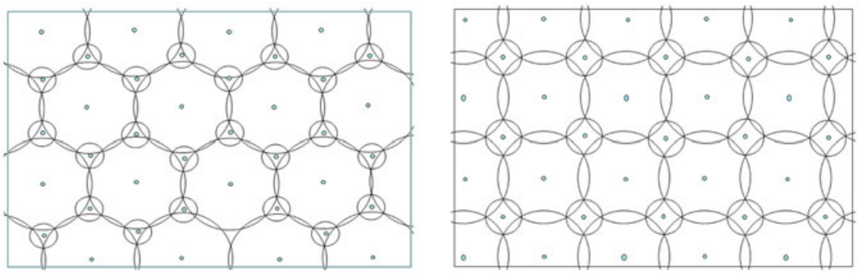

Figure 4. Plane cover models A-2 and B-2.

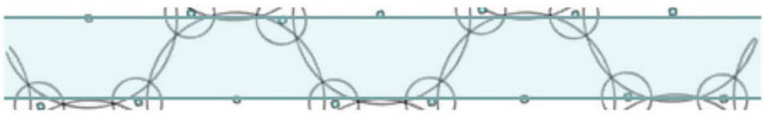

Figure 5. Outer band coverage Model SA-2.

Model A-2 generates the only efficient Model SA-2 of the outer plane cover (Figure 5), and Model B-2 determines two Models SB-2(1) and SB-2(2) for two different options of the band coverage with the same density (Figure 6).

Model SA-2: density $D_{SA-2} = \frac{11\pi}{9\sqrt{3}} \approx 2.2168$, when $R = \frac{h\sqrt{31}}{3\sqrt{3}} \approx 1.0715h$, $r = \frac{h}{3\sqrt{3}} \approx 0.19245h$.
Model SB-2(1): density $D_{SB-2(1)} = \frac{3\pi}{4} \approx 2.3562$, when $R = \frac{h\sqrt{5}}{2} \approx 1.1180h, r = \frac{h}{2}$.
Model SB-2(2): density $D_{SB-2(2)} = \frac{3\pi}{4} \approx 2.3562$, when $R = \frac{h\sqrt{5}}{2\sqrt{2}} \approx 0.7906h$, $r = \frac{h}{2\sqrt{2}} \approx 0.3536h$.

We performed calculations for Models A-2 and SA-2 with a minimal fragment (Figure 7).

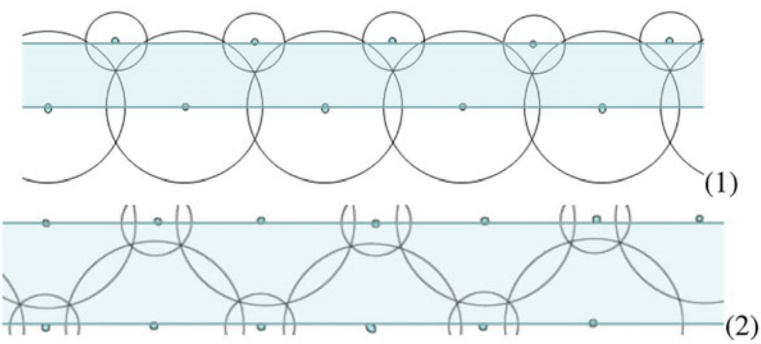

Figure 6. Outer band coverage models SB-2(1) and SB-2(2).

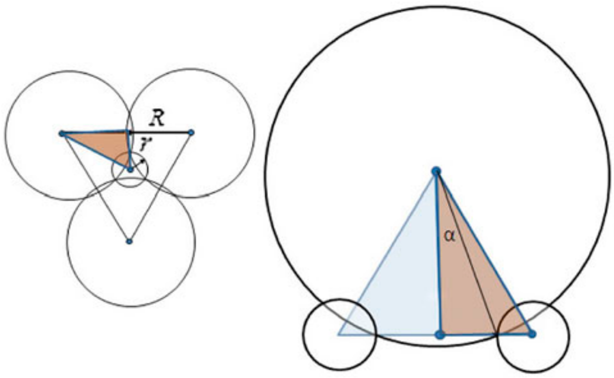

Figure 7. Fragments of models A-3 and SA-3 from the class of $\mathrm{Cov_F}$ (2: 1_{12}, 1_6).

Minimal fragment F is the right-angled triangle with acute angles of 30 and 60 degrees. If the hypotenuse equals to a and angle α determines the relative position of the cover discs, then

$$\mu(F) = \frac{a^2\sqrt{3}}{8}, \quad \mu(S) = \frac{\pi \cdot R^2}{12} + \frac{\pi \cdot r^2}{6},$$

where $R = \frac{a\sqrt{3}}{2\cos\alpha}$, $r = \frac{a}{2}\left(1 - \sqrt{3}\cdot\tan\alpha\right)$. After the necessary calculations we obtain the correlation of the cover density:

$$D(\alpha)_{A-2} = \frac{\mu(S)}{\mu(F)} = \frac{\pi}{6\sqrt{3}}(5 + 9\cdot\tan^2\alpha - 4\sqrt{3}\cdot\tan\alpha) = \frac{\pi}{6\sqrt{3}}\left(\left(3\cdot\tan\alpha - \frac{2}{\sqrt{3}}\right)^2 + \frac{11}{3}\right).$$

Therefore,

$$\min D_{A-2} = \frac{11\pi}{18\sqrt{3}} \approx 1.108433, \quad \min D_{SA-2} = \frac{11\pi}{9\sqrt{3}} \approx 2.216866, \quad \tan\alpha = \frac{2}{3\sqrt{3}}$$

Remark 1 Designation of the cover $\mathrm{Cov_F}$(2: 1_{12}, 1_6) allows one to determine an average correlation between the numbers of discs with different dimensions: $k_1 : k_2 = \frac{1}{12} : \frac{1}{6} = 1:2$. Thus, two small discs fall on one large disc. For other classes of covers, this procedure is correct as well.

5. Band coverage with discs of three radii

We consider a special plane cover with discs of three radii. In Figure 8, there are elements of this cover and its minimal fragment: Model A-3 of class $\mathrm{Cov_F}$(3: 1_4, 1_2, 1_2). It is clear that such plane cover determines the Model SA-3 of the outer band cover using discs of three different radii. The specific feature of the model is that the fragment dimension (correlation between the right triangle legs) appears to be an optimization parameter as well. Note that large discs have the same radius but, at optimal cover

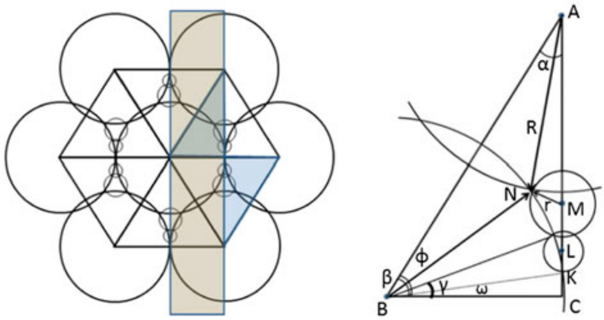

Figure 8. Elements of the plane cover with discs of three radii and the structure of its minimal fragment $\mathrm{Cov_F}(3\colon 1_4, 1_2, 1_2)$.

parameters, the hexagon shown in Figure 8 is slightly stretched across. Considering designations of the fragment, it means that the following correlations for the angles are performed: $\alpha > 30°, \alpha + \beta = 90°$.

We perform calculations and provide necessary explanations. For convenience, consider side BC equal to a (for a band it will be width h). Then, $AC = a/\tan\alpha$, $AB = a/\sin\alpha = 2R\cos\varphi$. From the latter correlation, we obtain the expression for the large radius: $R = \frac{a}{2\sin\alpha\cos\varphi}$.

From the triangle BCK, we obtain that cos $\omega = a/R$. Therefore,

$$\omega = \arccos\frac{a}{R} = \arccos(2\sin\alpha\cos\varphi).$$

If L is a small disc centre, then $\angle CBL = \omega + (\gamma - \omega)/2 = (\omega + \gamma)/2$. Further, we determine a small radius:

$$\rho = a \cdot tg(\angle CBL) - a \cdot \tan\omega = a\left(\tan\left(\frac{\omega+\gamma}{2}\right) - \tan\omega\right).$$

We find an expression for the middle radius r, as a distance between points M and N. It is easily seen that

$$x_M = a, \quad y_M = a \cdot \tan(\angle CBM) = a \cdot \tan\left(\frac{\beta - \varphi}{2} + \gamma\right) = a \cdot \tan\left(\frac{90° - \alpha - \varphi}{2} + \gamma\right),$$

$$x_N = R\cos(\beta - \varphi) = R\cos(90° - \alpha - \varphi) = \frac{a}{2\sin\alpha\cos\varphi}\sin(\alpha + \varphi) = \frac{a}{2}\left(1 + \frac{\tan\varphi}{\tan\alpha}\right),$$

$$y_N = R \cdot \tan(\beta - \varphi) = R/\tan(\alpha + \varphi) = \frac{a}{2} \cdot \frac{1}{\sin\alpha\cos\varphi\ \tan(\alpha + \varphi)},$$

$$r^2 = a^2\left(\left(\frac{\tan\alpha \cdot \tan\varphi - 1}{2\tan\alpha \cdot \tan\varphi}\right)^2 + \left(\tan\left(\frac{90° - \alpha - \varphi}{2} + \gamma\right) - \frac{1}{2\sin\alpha\cos\varphi\tan(\alpha + \varphi)}\right)^2\right).$$

Considering that $\mu(S) = \frac{\pi R^2}{4} + \frac{\pi r^2}{2} + \frac{\pi \rho^2}{2}, \mu(F) = \frac{a^2}{2\tan\alpha}$, we obtain the expressions for the cover density: $D(\alpha, \varphi, \gamma) = D_1 + D_2 + D_3$, where

$$D_1 = \frac{\pi \cdot \tan\alpha}{8(\sin\alpha\cos\varphi)^2},$$

$$D_2 = \pi \cdot \tan\alpha \cdot \left(\left(\frac{\tan\alpha \cdot \tan\varphi - 1}{2\tan\alpha \cdot \tan\varphi} \right)^2 + \left(\tan\left(\frac{90^\circ - \alpha - \varphi}{2} + \gamma \right) - \frac{1}{2\sin\alpha\cos\varphi\tan(\alpha + \varphi)} \right)^2 \right),$$

$$D_3 = \pi \cdot \tan\alpha \cdot \left(\tan\left(\frac{\omega + \gamma}{2} \right) - \tan\omega \right)^2, \quad \omega = \arccos(2\sin\alpha \cdot \cos\varphi).$$

Having determined the minimum, we obtain the following optimal parameters for the cover density function:

$$\min D_{A-3} \approx 1.0928276, \quad \min D_{SA-4} \approx 2.185655, \quad \text{and} \quad \alpha \approx 29.79^\circ, \quad \beta \approx 60.21^\circ, \quad R \approx 1.0644a, \quad r \approx 0.2003a, \quad \rho \approx 0.0305a.$$

Consider one more plane cover with discs of three radii: Model B-3 of class $\mathrm{Cov_F}(3\colon 1_4, 1_4, 1_2)$. In Figure 9, there are elements of this cover and its minimal fragment being a rectangle with the shape close to the square. The structural properties of this cover also allow one to determine the Model SB-3 of the outer band cover using discs of three different radii.

We perform calculations using the designations in Figure 9. From the right triangle with angle α, we obtain $R = h/\cos\alpha$. On the analogy, from the right triangle with angle β, obtain the expression for the side of a rectangle: $\mathrm{AD} = R \cdot \cos\beta = h \cdot \cos\beta / \cos\alpha$. Hence, we may determine the fragment area: $\mu(F) = h^2 \cos\beta / \cos\alpha$.

Further calculations for two other radii are:

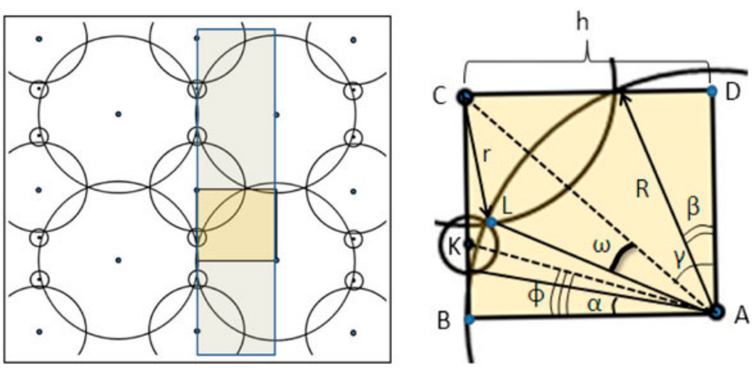

Figure 9. Plane cover elements with the discs of three radii and the structure of its minimal fragment $\mathrm{Cov_F}(3\colon 1_4, 1_4, 1_2)$.

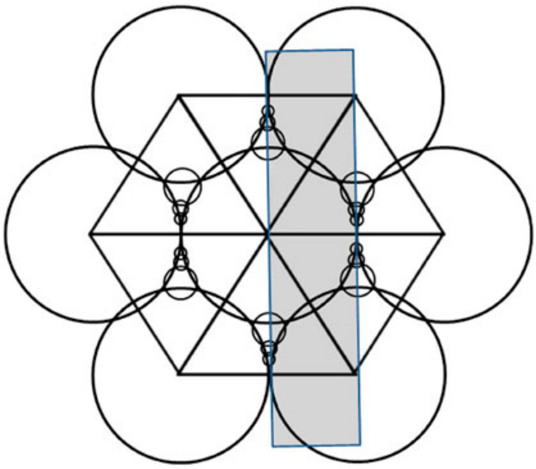

Figure 10. Element of plane and band cover with discs of four $\mathrm{Cov_F}(4{:}\ 1_4, 1_2, 1_2, 1_2)$.

$$r = h - R \cdot \sin\beta = h(1 - \sin\beta/\cos\alpha), \quad \rho = h \cdot \tan\varphi - h \cdot \tan\alpha = h \cdot (\tan\varphi - \tan\alpha).$$

It may be shown that $\varphi = 45^\circ + \frac{\alpha+\beta}{2} - \gamma = 45^\circ + \frac{\alpha+\beta}{2} - \arctan\left(\frac{\cos\alpha}{\cos\beta}\right)$.

For the area of the fragment cover, we obtain:

$$\mu(S) = \pi\left(\frac{R^2}{4} + \frac{r^2}{4} + \frac{\rho^2}{2}\right) = \frac{h^2\pi}{4}\left(\frac{1}{\cos^2\alpha} + \left(1 - \frac{\sin\beta}{\cos\alpha}\right)^2 + 2\left(\tan\left(45^\circ + \frac{\alpha+\beta}{2} - \arctan\left(\frac{\cos\alpha}{\cos\beta}\right)\right) - \tan\alpha\right)^2\right).$$

This allows one to write the cover density function:

$$D(\alpha,\beta) = \frac{\mu(S)}{\mu(F)} = \frac{\pi \cdot \cos\alpha}{4\cos\beta}\left(\frac{1}{\cos^2\alpha} + \left(1 - \frac{\sin\beta}{\cos\alpha}\right)^2 + 2\left(\tan\left(45^\circ + \frac{\alpha+\beta}{2} - \arctan\left(\frac{\cos\alpha}{\cos\beta}\right)\right) - \tan\alpha\right)^2\right).$$

Having determined the minimum for the function of the cover density, it is possible to obtain the following optimal parameters:

$$\min D_{B-3} \approx 1.15453, \quad \min D_{SB-3} \approx 2.30906 \quad \text{and} \quad \alpha \approx 16.56^0, \quad \beta \approx 30.4221^\circ,$$
$$R \approx 1.043265h, \quad r \approx 0.471695h, \quad \rho \approx 0.075886h.$$

Remark 2 Comparing the built models of the outer band covers, it may be noticed that, adding a new disc size, we use the generalization of the previous models. Therefore, it may be stated that we have considered all rational covers with discs of one, two, and three radii. If the models with a large number of different discs are built, the overall complexity grows (Figure 10), but there is an insignificant decrease of density.

Table 1. Comparison of the coverage models characteristics.

No.	Band cover model	Coverage class	Minimal cover density	Disc radii	Correlation of the number of discs (k_1:k_2) or (k_1:k_2:k_3)
1	SA-1(1)	$\mathrm{Cov_F}(1{:}\ 1_{12})$	$D \approx 2.4184$	$R = 2\ h/3$	–
2	SA-1(2)	$\mathrm{Cov_F}(1{:}\ 1_{12})$	$D \approx 2.4184$	$R = 2\ h$	–
3	SB-2(1)	$\mathrm{Cov_F}(2{:}\ 1_4, 1_4)$	$D \approx 2.3562$	$R \approx 1.12\ h,\ r = 0.5\ h$	1:1
4	SB-2(2)	$\mathrm{Cov_F}(2{:}\ 1_4, 1_4)$	$D \approx 2.3562$	$R \approx 0.79\ h,\ r \approx 0.35\ h$	1:1
5	SA-2	$\mathrm{Cov_F}(2{:}\ 1_{12}, 1_6)$	$D \approx 2.2169$	$R \approx 1.08\ h,\ r \approx 0.19\ h$	1:2
6	SA-3	$\mathrm{Cov_F}(3{:}\ 1_4, 1_2, 1_2)$	$D \approx 2.18366$	$R \approx 1.06\ h,\ r \approx 0.20\ h,\ p \approx 0.03\ h$	1:2:2
7	SB-3	$\mathrm{Cov_F}(3{:}\ 1_4, 1_4, 1_2)$	$D \approx 2.30906$	$R \approx 1.04\ h,\ r \approx 0.47\ h,\ p \approx 0.076\ h$	1:1:2

6. Conclusion and outlook

Several outer band cover models with the best parameters for their class are considered in the paper. The problem of the areas outer coverage is closely related to the plane coverage. To some extent, it has its own specificity, and a search for efficient covers variants is performed in a more 'compact' logical space. We may clearly specify all possible options of a certain class of covers for the mentioned types of fragments and choose the best ones.

Below, there is a Table 1 of results providing more obvious comparison of the characteristics of the outer band disc cover models.

In the future, we are planning to consider the construction of min-density regular outer band covers using ellipses in a three-dimensional space. This generalization of the problem can be proved, for example, by the following consideration. If a sensor is equipped with a video camera that is located above the surface and that views the surface, then the covered region is an ellipse, and its shape depends on such parameters as object glass height, focus and incidence. It is evident that some covers, using ellipses, can be constructed from the covers that use discs applying the affine transformation (AT). An AT is a transformation that preserves the coverage density. Examples of ATs include compression. Suppose we know a small density band cover with discs C, when the disks centres are located in any places (inside the band, too).[7] Applying an AT (suitable compression along the band) to cover C, we obtain a cover with ellipses, which are generated by the sensors located on the border of the band above the surface (the projection of the sensor location reaches the border).

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